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A new family of filtration seven in the stable homotopy of spheres

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ABSTRACT. This paper proves the existence of a new family of nontrivial homotopy elements in the stable homotopy of spheres which is of degree $2(p-1)(p^n+3p^2+3p+3)-7$ and is represented by $b_{n-1}g_0\gamma_3$ in the $E_2^{7,*}$-term of the Adams spectral sequence, where $p \geqslant 7$ is a prime and $n \geqslant 4$. In the course of proof, a new family of homotopy elements in $\pi_* V(1)$ which is represented by $b_{n-1}g_0$ in the $E_2^{4,*}V(1)$-term of the Adams spectral sequence is detected.

1. Introduction

Let A be the mod p Steenrod algebra and S the sphere spectrum localized at an odd prime p. To determine the stable homotopy groups of spheres $\pi_* S$ is one of the central problems in homotopy theory. One of the main tools to reach it is the Adams spectral sequence (ASS) $E_2^{s,t} = \mathrm{Ext}_A^{s,t}(Z_p, Z_p) \Rightarrow \pi_{t-s}S$, where the $E_2^{s,t}$-term is the cohomology of A. If a family of generators x_i in $E_2^{s,*}$ converges nontrivially in the ASS, then we get a family of homotopy elements f_i in $\pi_* S$ and we say that f_i is represented by $x_i \in E_2^{s,*}$ and has filtration s in the ASS. So far, not so many families of homotopy elements in $\pi_* S$ have been detected. For example, a family $\zeta_{n-1} \in \pi_{p^nq+q-3}S$ for $n \geqslant 2$ which has filtration 3 and is represented by $h_0 b_{n-1} \in \mathrm{Ext}_A^{3,p^nq+q}(Z_p, Z_p)$ has been detected in [2], where $q = 2(p-1)$. The main purpose of this paper is to detect a new family of homotopy elements in $\pi_* S$ which has filtration 7 in the ASS.

From [3], $\mathrm{Ext}_A^{1,*}(Z_p, Z_p)$ has Z_p-base consisting of $a_0 \in \mathrm{Ext}_A^{1,1}(Z_p, Z_p)$, $h_i \in \mathrm{Ext}_A^{1,p^iq}(Z_p, Z_p)$ for all $i \geqslant 0$ and $\mathrm{Ext}_A^{2,*}(Z_p, Z_p)$ has Z_p-base consisting of α_2, a_0^2, $a_0 h_i$ $(i > 0)$, g_i $(i \geqslant 0)$, k_i $(i \geqslant 0)$, b_i $(i \geqslant 0)$, and $h_i h_j$ $(j \geqslant i+2, i \geqslant 0)$ whose internal degree are $2q+1$, 2, p^iq+1, $p^{i+1}q+2p^iq$, $2p^{i+1}q+p^iq$, $p^{i+1}q$ and p^iq+p^jq respectively. From [1] p.110 table 8.1, there is a generator $\gamma_3 \in \mathrm{Ext}_A^{3,(3p^2+2p+1)q}(Z_p, Z_p)$ whose name in [1] is $h_{0,1,2,3}$. Our main result is the following theorem.

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