HIROSHIMA MATH. J.
28 (1998), 501–506

Liouville-Picard theorem in harmonic spaces

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(Received October 6, 1997)

ABSTRACT. An extended version of the classical Liouville-Picard theorem for the harmonic functions in $\mathbb{R}^n$ is considered in the context of biharmonic functions in a Brelot harmonic space with a symmetric Green kernel.

1. Introduction

In [3] is considered a Liouville-Picard type theorem for superharmonic functions in $\mathbb{R}^n$, $n \geq 2$. A simple special case of this theorem shows that if $s \geq 0$ is superharmonic in $\mathbb{R}^n$, $n = 3$ or 4, and $\Delta^2 s \geq 0$ then s is a constant and this result is not true if $n \geq 5$.

Since for a superharmonic function s in a domain ω in $\mathbb{R}^n$, $n \geq 2$, the condition $\Delta^2 s \geq 0$ is equivalent to saying that Δs is subharmonic ≤ 0 in ω, the above special case can be formulated as follows: there exist p and q, potentials > 0 in $\mathbb{R}^n$ such that $\Delta q = -p$ if and only if $n \geq 5$. This shows a variation in the study of potential theory in $\mathbb{R}^n$, $n \geq 3$, depending on n, even though (symmetric) Green kernels can be defined in all these spaces.

In this note, we obtain some results which reflect this variation. With a view to introduce only the essential assumptions in the proofs, we have chosen to work in a Brelot harmonic space possessing a symmetric Green kernel [2]. Another advantage is that some of these results, proved earlier in a Riemannian manifold [5] but not meaningful in a Riemann surface because the Laplacian is not invariant under a parametric change, have a general validity.

2. Preliminaries

Let Ω be a Brelot harmonic space with a countable base, having potentials > 0 and satisfying the axiom of proportionality; then, Mme. R. M. Hervé has proved that there exists a Green function $G(x, y)$ on Ω which is assumed here to be symmetric; it is also assumed that the constants are harmonic in Ω. (The terms are explained in F. Y. Maeda [2], p. 35).

1991 *Mathematics subject classification:* 31D05
Key words and phrases: Liouville-Picard theorem; bipotential spaces.