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A note on spaces of test and generalized functions of Poisson white noise

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Abstract. The paper is devoted to construction and investigation of some riggings of the L^2-space of Poisson white noise. A particular attention is paid to the questions whether the test space consists of functions with continuous version and whether the test space is algebra under pointwise multiplication of functions.

1. Introduction

In the works [5, 6], started a study of test and generalized functions defined on the Schwartz space of tempered distributions $\mathscr{S}'(\mathbb{R})$ the dual pairing of which is determined by the inner product of the L^2-space $(L^2_{\mathrm{P}}) = L^2(S'(\mathbb{R}), d\mu_{\mathrm{P}})$, where μ_{P} is the measure of Poisson white noise. Following the construction of the space of Hida distributions in Gaussian analysis (e.g., [10, 2, 4, 19]), Ito and Kubo [6] introduced the triple $(S_{\mathrm{P}})^* \supset (L^2_{\mathrm{P}}) \supset (S_{\mathrm{P}})$. However, the following two important problems remained open: 1) Does the space (S_{P}) consist of continuous functions, or, which is "almost" equivalent, do the delta functions belong to $(S_{\mathrm{P}})^*$? 2) Is the space (S_{P}) an algebra under pointwise multiplication of functions?

In this note, we will construct a whole scale of test spaces $(S_{\mathrm{P}})^{\varkappa}$, $\varkappa \geq 0$, such that $(S_{\mathrm{P}})^{\varkappa_1} \subset (S_{\mathrm{P}})^{\varkappa_2}$ if $\varkappa_1 > \varkappa_2$, and of their dual spaces $(S_{\mathrm{P}})^{-\varkappa}$ with respect to (L^2_{P}). For $\varkappa = 0$, $(S_{\mathrm{P}})^0 = (S_{\mathrm{P}})$, so that $(S_{\mathrm{P}})^{-0} = (S_{\mathrm{P}})^*$. The idea of construction of these spaces comes from the corresponding constructions in Gaussian analysis [8, 7].

The main results of the paper are as follows: 1) The space $(S_{\mathrm{P}})^1$ consists of continuous functions, and for each $(S_{\mathrm{P}})^{\varkappa}$ with $\varkappa < 1$ this is not the case. 2) The space $(S_{\mathrm{P}})^1$ (and even each space $(S_{\mathrm{P}})^{\varkappa}$ with $\varkappa > 1$) is an algebra under the continuous pointwise multiplication. The estimate of Hilbert norms are analogous to the estimates in Gaussian analysis, e.g., [11, 20, 21, 4].

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