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# Nonexistence of subsolutions of a nonlinear elliptic equation on bounded domains in a Riemannian manifold

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ABSTRACT. We give some nonexistence results for positive subsolutions of a certain class of nonlinear elliptic equations including the scalar curvature equation on bounded domains in a Riemannian manifold.

## 1. Introduction

Let $(M,g)$ be a Riemannian manifold ($n=\dim M \geq 3$), $S_g$ its scalar curvature. For any smooth function $f$ on $M$, $f$ can be realized as the scalar curvature of some metric $\tilde{g}$ conformal to $g$, if and only if there exists a smooth solution $u$ of the following equation:

$$(*) \qquad \begin{cases} -\frac{4(n-1)}{n-2}\Delta_g u + S_g u = f u^{(n+2)/(n-2)} & \text{on} \quad M, \\ u > 0 \end{cases}$$

where $\Delta_g$ is the Laplacian of $g$ (i.e. $\Delta_g := g^{ij}\nabla_{ij}$). Indeed, the conformal metric $\tilde{g} = u^{4/(n-2)}g$ has the scalar curvature $S_{\tilde{g}} = f$.

In this paper, we consider a class of equations including $(*)$ on a certain type of open Riemannian manifolds, and give some nonexistence results in the case $f$ is nonpositive. Before describing our results, we recall here some known facts for typical $(M,g)$'s. Throughout this paper, we use the notation "$f \sim \tilde{f}$" to mean that $f/\tilde{f}$ is bounded between two positive constants (i.e. $C\tilde{f} \leq f \leq C'\tilde{f}$ for some $C > 0$ and $C' > 0$).

FACT 1.1. *Let $(M,g)$ be the Euclidean space $(\mathbf{R}^n, g_0)$. Denote the distance function to the origin by $r$. Then the following assertions hold:*

(1) *If $|f| \leq Cr^{-2-\varepsilon}$ near $\infty$ for some $C > 0$ and $\varepsilon > 0$, then $(*)$ has infinitely many solutions $u$ satisfying $u \sim 1 = r^0$ near $\infty$* ([14]);

(2) *If $-Cr^{-2-\varepsilon} \leq f < 0$ near $\infty$ for some $C > 0$ and $\varepsilon > 0$, and $f \leq 0$ on $\mathbf{R}^n$, then $(*)$ has a solution $u$ satisfying $u \geq C'r^{\varepsilon(n-2)/4}$ near $\infty$ for some $C' > 0$* ([4]);

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