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On localized weak precompactness in Banach spaces II

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ABSTRACT. This is a supplement and continuation of our previous paper [9], in which we have made a study of K-weakly precompact sets in Banach spaces. For a bounded subset A of dual Banach spaces, we introduce notions of A-separated δ-trees, A-midpoint-Bocce-dentability, A-strong regularity and A-weak*-strong regularity. Making use of these notions and arguments analogous to that of [9], we give some more characterizations of K-weakly precompact sets.

1. Introduction

Throughout this paper, X denotes an arbitrary real Banach space, X^* and X^{**} its topological dual space and bidual space, respectively, and $B(X)$ (resp. $S(X)$) the closed unit ball (resp. sphere) of X. The triple (I, Λ, λ) refers to the Lebesgue measure space on I $(= [0,1])$, Λ^+ to the sets in Λ with positive measure, L_1 to $L_1(I, \Lambda, \lambda)$ and L_∞ to $L_\infty(I, \Lambda, \lambda)$. For each $B \in \Lambda^+$, denote $\Delta(B) = \{\chi_F/\lambda(F) : F \subset B, F \in \Lambda^+\}$. For each $g \in L_\infty$ and $B \in \Lambda^+$, ess-O $(g|B)$ denotes the essential oscillation of g (as a function) on B. We always understand that I is endowed with Λ and λ. If C is a subset of X^{**}, a function $f : I \to X^*$ is said to be C-measurable if the real-valued function $(x^{**}, f(t))$ is λ-measurable for each $x^{**} \in C$. If $C = X$, we say that f is weak*-measurable. If $f : I \to X^*$ is a bounded weak*-measurable function, we obtain a bounded linear operator $T_f : X \to L_1$ given by $T_f(x) = x \circ f$ for every $x \in X$, where $(x \circ f)(t) = (x, f(t))$ for every $t \in I$. The dual operator of T_f is denoted by T_f^* $(: L_\infty \to X^*)$. Furthermore, if we define a vector measure $\alpha_f : \Lambda \to X^*$ by $\alpha_f(B) = T_f^*(\chi_B)$ for every $B \in \Lambda$, we then have that

$$(x, \alpha_f(B)) = \int_B (x, f(t))\, d\lambda(t)$$

for every $x \in X$ and every $B \in \Lambda$. Let $\{I(n,i) : n = 0, 1, \ldots; i = 0, \ldots, 2^n - 1\}$ be a system of intervals in I given by $I(n,i) = [i/2^n, (i+1)/2^n)$ if $n \geqq 1$, $0 \leqq i \leqq 2^n - 2$ and $I(n, 2^n - 1) = [(2^n - 1)/2^n, 1]$ if $n \geqq 0$. If Λ_n denotes the

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