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Geometry of contrast functions and conformal geometry

Hiroshi MATSUZOE

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ABSTRACT. We give a necessary and sufficient condition for a pseudo-Riemannian manifold with a compatible affine connection to be projectively flat, dual-projectively flat or conformally flat in terms of the Batlett tensor, which is derived from forth-order derivatives of contrast function.

Introduction

In 1982, Nagaoka and Amari formulated information geometry, which has been applied to various fields of information sciences, for example, information theory, neural networks, system theory, and so on (cf. [2]). In information geometry, contrast functions play an essential role. The Kullback-Leibler divergence is an interesting example of contrast function, which is used in statistical inference (see, [1]). A geometric divergence introduced by Kurose [8] is also an example of contrast function. In this paper, we study the geometry of geometric divergences in affine differential geometry.

In general, a contrast function ρ induces a dualistic geometrical structure on M (cf. [4] and [8]). The second-order derivatives of a given contrast function ρ of M induce a pseudo-Riemannian metric h on M. The third-order derivatives induce two torsion-free affine connections ∇ and ∇^* such that these are mutually dual with respect to h. In this case, the tensors ∇h and $\nabla^* h$ are symmetric. Then the triplets (M, ∇, h) and (M, ∇^*, h) are statistical manifolds.

The *Bartlett tensor* B of contrast function ρ, which was formulated by Eguchi [4], is defined from forth-order derivatives of ρ. The anti-symmetric part of B is the curvature tensor of the induced affine connection ∇. The Bartlett tensor B and the dual Bartlett tensor B^* correspond to the Bartlett corrections in likelihood ratio tests in statistics.

In this paper, we study the Bartlett tensors of contrast functions in the geometric divergences case. As applications, we give necessary and sufficient conditions for a statistical manifold to be projectively flat, dual-projectively flat or conformally flat in terms of the Bartlett tensor.

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