HIROSHIMA MATH. J.
29 (1999), 145–160

Self-similar radial solutions to a parabolic system modelling chemotaxis via variational method

Yutaka MIZUTANI, Naomi MURAMOTO and Kiyoshi YOSHIDA

(Received August 29, 1997)
(Revised March 11, 1998)

1. Introduction

In the previous paper [2] the first author studied the positive self-similar radial solutions

$$u(x,t) = \frac{1}{t}\varphi\left(\frac{|x|}{\sqrt{t}}\right), \quad v(x,t) = \psi\left(\frac{|x|}{\sqrt{t}}\right)$$

concerning the system of parabolic differential equations

$$\text{(KS)} \qquad \begin{cases} \dfrac{\partial u}{\partial t} = \nabla \cdot (\nabla u - \chi u \nabla v) & \text{in } \boldsymbol{R}^2, \quad t > 0, \\ \varepsilon \dfrac{\partial v}{\partial t} = \Delta v + \alpha u & \text{in } \boldsymbol{R}^2, \quad t > 0, \end{cases}$$

where α, χ and ε are positive constants. This system is one of the mathematical model by [1] describing chemotactic aggregation of cellular slime molds which move preferentially towards relatively high concentrations of a chemical substance secreted by the amoebae themselves. At place x and time t, $u(x,t)$ means the cell density of the cellular slime molds, and $v(x,t)$ the concentration of the chemical substance. Substitute $u = \varphi/t$ and $v = \varphi$ in (KS) and note φ and ψ are radially symmetric in x. Then $(\varphi(r), \psi(r))$ with $r = |x|/\sqrt{t}$ satisfies

$$\text{(KSO)} \qquad \begin{cases} (\varphi' - \chi\varphi\psi')' + \dfrac{1}{r}(\varphi' - \chi\varphi\psi') + \dfrac{r}{2}\varphi' + \varphi = 0 \\ \psi'' + \dfrac{1}{r}\psi' + \dfrac{\varepsilon r}{2}\psi' + \alpha\varphi = 0 \\ \varphi'(0) = \psi'(0) = 0. \end{cases}$$

From the first equation in (KSO) we have

$$\{2r(\varphi' - \chi\varphi\psi') + r^2\varphi\}' = 0 \quad \text{for } r > 0,$$

1991 *Mathematics Subject Classification*: 35Q80, 92C46.
Key words and phrases: Chemotaxis, self-similar radial solution.