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Asymptotic analysis of a phase field model with memory for vanishing time relaxation

Pierluigi COLLI, Gianni GILARDI and Maurizio GRASSELLI

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ABSTRACT. A phase-field model accounting for memory effects is considered. This model consists of a hyperbolic integrodifferential equation coupled with a parabolic differential inclusion. The latter relation rules the evolution of the phase field and contains a time relaxation parameter which happens to be very small in the applications. A well-posed initial and boundary value problem for the evolution system is introduced and the asymptotic behavior of its solution as the time relaxation goes to zero is analyzed rigorously. Convergence results and error estimates are obtained under suitable assumptions ensuring that the limit problem has a unique solution.

1. Introduction

Consider a two-phase system which occupies a bounded domain $\Omega \subset \mathbf{R}^3$ until a given time $T > 0$. Denote by ϑ its relative temperature (fixed in order that $\vartheta = 0$ is the equilibrium temperature between the two phases) and by χ the so-called phase-field, that is, an order parameter which could represent the local proportion of one phase. To describe the evolution of the pair (ϑ, χ), we have recently introduced and studied the following system (see [7–9])

$$\partial_t(\vartheta + \lambda\chi + \varphi * \vartheta + \psi * \chi) - \Delta(k * \vartheta) = g \quad \text{in } \Omega \times (0, T) \tag{1.1}$$

$$\mu\partial_t\chi - \nu\Delta\chi + \beta(\chi) \ni \gamma(\chi) + \lambda\vartheta \quad \text{in } \Omega \times (0, T) \tag{1.2}$$

coupled with the boundary and initial conditions

$$\partial_n(k * \vartheta) = h \quad \text{and} \quad \partial_n\chi = 0 \quad \text{on } \partial\Omega \times (0, T) \tag{1.3}$$

$$\vartheta(0) = \vartheta_0 \quad \text{and} \quad \chi(0) = \chi_0 \quad \text{in } \Omega. \tag{1.4}$$

Here $*$ denotes the usual time convolution product over $(0, T)$, defined by

$$(a * b)(t) = \int_0^t a(s)b(t-s)\,ds, \quad t \in [0, T] \tag{1.5}$$

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