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Minimax estimation of common variance in normal distributions when the mean vector is known to lie in an ellipsoid

Teruo FUJIOKA

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ABSTRACT. This paper is concerned with minimax estimation of variance when n samples $y_1, \dots, y_n$ are independently normally distributed with common variance. Here it is assumed that $(E(y_1), \dots, E(y_n))$ is known to lie in an ellipsoid. A new class of estimators which are quadratic in $y_1, \dots, y_n$ are introduced and the minimax estimators are explicitly given. The case of i.i.d. sample with $N(0, \sigma^2)$ is discussed as a special case where the ellipsoid degenerates to the origin. In this case our minimax estimator provides the minimum mean squared error estimator of σ^2.

1. Introduction

This paper is concerned with minimax estimation of variance in a model which is closely related to a nonparametric regression. We consider a simplified model. Let y_i $(i = 1, \dots, n)$ be independently distributed as $N(\mu_i, \sigma^2)$, where both the mean vector $(\mu_1, \dots, \mu_n)$ and the variance σ^2 are unknown. The mean vector is assumed to lie in an ellipsoid

$$\sum_{i=1}^{n} \lambda_i \mu_i^2 \leq r\sigma^2 \tag{1}$$

with fixed constants $0 < \lambda_1 < \cdots < \lambda_n$ and a fixed value $r > 0$. Speckman [21] introduced such a model by considering a simplified formulation of spline smoothing in nonparametric regression. Let the observation y_i be taken at a design point $t_i \in [a, b]$. Suppose that $y_i = f(t_i) + \varepsilon_i$, where f is a smooth function, and ε_i is distributed with mean 0 and unknown variance σ^2. It is assumed that f has a bounded square integrable qth derivative, and a squared norm for f is defined by $\|f^{(q)}\|^2 = \int_a^b |f^{(q)}(t)|^2 \, dt$. Let $\mathscr{S}_n^q$ be the space of natural polynomial splines of degree $2q - 1$ with knots $\{t_1, \dots, t_n\}$, and $\{\varphi_1, \dots, \varphi_n\}$ be the basis introduced by Demmler-Reinsch [6]. If $f = \sum \beta_k \varphi_k \in$

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