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The distribution of zeros of solutions of neutral differential equations

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ABSTRACT. In this paper we establish an estimate for the distance between adjacent zeros of the oscillatory solutions of the neutral delay differential equation $[x(t)+P(t)x(t-\tau)]'+Q(t)x(t-\sigma)=0$, where $P, Q \in C([t_0,\infty),\mathbf{R}^+)$ and $\tau,\sigma \in \mathbf{R}^+$.

1. Introduction

Consider the first order neutral delay differential equation

$$[x(t)+P(t)x(t-\tau)]'+Q(t)x(t-\sigma)=0 \tag{1}$$

where

$$P \in C([t_0,\infty),[0,\infty)), \qquad Q \in C([t_0,\infty),(0,\infty)), \qquad \sigma > \tau > 0. \tag{2}$$

When $P(t) \equiv 0$, Eq.(1) reduces to

$$x'(t)+Q(t)x(t-\sigma)=0. \tag{3}$$

The oscillation theory of neutral differential equations has been extensively developed during the past several years. We refer to the monographs by Györi and Ladas [1], Bainov and Mishev [2], Erbe, Kong and Zhang [3], and the references cited therein. But the results dealing with the distribution of zeros of the oscillatory solution of neutral differential equation are relatively scarce. Recently, Erbe et al. [3] and Liang [4] established estimates for the distance between adjacent zeroes of the solutions of Eq.(3). Zhou and Wang [5] extend the results in [3]. In this paper, by using a new technique, we establish an estimate for the distance between adjacent zeroes of the solutions of Eq.(1). Our results improve the known results in [3–5].

Let $m=\max\{\tau,\sigma\}$. By a solution of Eq.(1) we mean a function $x \in C([t_x-m,\infty),\mathbf{R})$, for some $t_x \geq t_0$, such that $x(t)+P(t)x(t-\tau)$ is continuously differentiable on $[t_x,\infty)$ and such that Eq.(1) is satisfied for $t \geq t_x$.

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