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Some homotopy groups of the rotation group R_n

Dedicated to Professor Teiich Kobayashi on his 60th birthday

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ABSTRACT. We determine the group strucures of the 2-primary components of the homotopy groups of the rotation group $\pi_k(R_n)$ for $k = 17$ and 18 by use of the fibration $R_{n+1}/R_n = S^n$.

Introduction

We denote by R_n the n-th rotation group. We know the homotopy groups $\pi_k(R_n)$ for $k \leq 15$ by [7]. According to [9] and [8], the group structures of $\pi_k(R_n)$ for $k \leq 22$ and $n \leq 9$ are known. For $k = 15$ and 16, we know the 2-primary components of $\pi_k(R_n)$ ([5]). We denote by $\pi_k(X:2)$ a suitablly chosen subgroup of the homotopy group $\pi_k(X)$ which consists of the 2-primary component and a free part such that the index $[\pi_k(X):\pi_k(X:2)]$ is odd. The purpose of the present note is to determine $\pi_k(R_n:2)$ for $k = 17$ and 18.

Our method is the composition methods developed by Toda [17]. We freely use generators and relations in the homotopy groups of spheres $\pi_{n+k}(S^n)$ for $k \leq 18$. In determining $\pi_{18}(R_n:2)$, the precise informations of the generators of $\pi_{n+18}(S^n)$ for $10 \leq n \leq 12$ ([14]) are essentially used. Our main tool is to use the following exact sequence induced from the fibration $R_{n+1}/R_n = S^n$:

$$(k)_n \qquad \pi_{k+1}(S^n) \xrightarrow{\Delta} \pi_k(R_n) \xrightarrow{i_*} \pi_k(R_{n+1}) \xrightarrow{p_*} \pi_k(S^n) \xrightarrow{\Delta} \pi_{k-1}(R_n),$$

where $i: R_n \hookrightarrow R_{n+1}$ is the inclusion, $p: R_{n+1} \to S^n$ is the projection and Δ is the connecting map.

The metastable range is obtained from the splitting ([2]):

$$\pi_k(R_n) \cong \pi_k(R_\infty) \oplus \pi_{k+1}(V_{2n,n}) \qquad \text{for } k \leq 2n-1 \quad \text{and} \quad n \geq 13,$$

where $V_{m,r} = R_m/R_{m-r}$ for $m \geq r$ is the Stiefel manifold.

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