HIROSHIMA MATH. J.
29 (1999), 313–325

On the existence of solutions of nonlinear boundary value problems at resonance in Sobolev spaces of fractional order

Thomas RUNST*

(Received April 10, 1998)
(Revised June 18, 1998)

ABSTRACT. The purpose of this paper is to prove existence results for a class of degenerate boundary value problems for second-order elliptic operators in the framework of Sobolev spaces of fractional order. The proofs apply generalized solvability conditions of Landesman-Lazer type, Leray-Schauder degree arguments and maximum principles.

1. Introduction and main result

Let $\Omega \subset \mathbf{R}^n$ be a bounded domain with C^∞ boundary $\partial\Omega$. Let

$$Au(x) = -\sum_{i=1}^{n} \frac{\partial}{\partial x_i} \left(\sum_{j=1}^{n} a_{ij}(x) \frac{\partial u}{\partial x_j}(x) \right) + c(x)u(x)$$

be a second order elliptic differential operator with real C^∞ functions a_{ij}, c on $\bar{\Omega}$ satisfying the following properties:

(p1) $a_{ij}(x) = a_{ji}(x),\ i, j = 1, \dots, n,\ x \in \bar{\Omega}$.

(p2) There exists a positive constant C_0 such that for all $x \in \bar{\Omega}$ and all $\xi \in \mathbf{R}^n$

$$\sum_{i,j=1}^{n} a_{ij}(x)\xi_i\xi_j \geq C_0|\xi|^2.$$

(p3) $c(x) \geq 0$ on $\bar{\Omega}$.

We consider the following class of degenerate boundary value problems for semilinear second-order elliptic differential operators

$$Au - \lambda_1 u = g(u) + f \quad \text{in } \Omega, \qquad Bu = a\frac{\partial u}{\partial \nu} + bu = 0 \quad \text{on } \partial\Omega \tag{P}$$

---

*partly supported by Deutsche Forschungsgemeinschaft, grant Tr 374/1-2
1991 *Mathematics Subject Classification.* 35J65, 47H11, 47H15.
Key words and phrases. Degenerate boundary value problems, Landesman-Lazer conditions, Leray-Schauder degree.