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A note on the localization of J-groups

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ABSTRACT. Let $\widetilde{JO}(X) = \widetilde{KO}(X)/TO(X)$ be the J-group of a connected finite CW complex X. Using Atiyah-Tall [5], we obtain two computable formulae of $TO(X)_{(p)}$, the localization of $TO(X)$ at a prime p. Then we show how to use those two formulae of $TO(X)_{(p)}$ to find the J-orders of elements of $\widetilde{KO}(X)$, at least the 2 and 3 primary factors of the canonical generators of $\widetilde{JO}(\mathbf{C}P^m)$. Here $\mathbf{C}P^m$ is the complex projective space.

1. Introduction

Let $\widetilde{JO}(X) = \widetilde{KO}(X)/TO(X)$ be the J-group of a connected finite CW complex X, where $\widetilde{KO}(X)$ is the additive subgroup of the KO-ring $KO(X)$ of elements of virtual dimension zero and $TO(X) = \{E - F \in \widetilde{KO}(X) : S(E \oplus n)$ is fibre homotopy equivalent to $S(F \oplus n)$ for some $n \in \mathbf{N}\}$. Let ψ^k be the Adams operations. Then Adams [1] and Quillen [13] showed that $TO(X) = WO(X) = VO(X)$. Here

$$WO(X) = \bigcap_f \widetilde{KSO}(X)_f \tag{1}$$

where the intersection runs over all functions $f : \mathbf{N} \to \mathbf{N}$ and $\widetilde{KSO}(X)_f = \langle k^{f(k)}(\psi^k - 1)(u) : u \in \widetilde{KSO}(X) \text{ and } k \in \mathbf{N}\rangle$, and

$$VO(X) = \left\{ x \in \widetilde{KSO}(X) : \text{there exists } u \in \widetilde{KSO}(X) \text{ such that} \right.$$
$$\left. \theta_k(x) = \frac{\psi^k(1+u)}{1+u} \in 1 + \widetilde{KSO}(X) \otimes \mathbf{Q}_k \text{ for all } k \in \mathbf{N} \right\} \tag{2}$$

where θ_k are the Bott exponential classes, and $\mathbf{Q}_k = \{n/k^m : n, m \in \mathbf{Z}\}$.

For a prime p, let $\widetilde{JO}(X)_{(p)}$ denote the localization of $\widetilde{JO}(X)$ at p. Since $\widetilde{JO}(X)$ is a finite abelian group, $\widetilde{JO}(X)_{(p)}$ is isomorphic to the p-summand of

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