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Representation of α-harmonic functions in Lipschitz domains

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ABSTRACT. We give the Martin representation for nonnegative functions which are harmonic on Lipschitz domains with respect to symmetric stable processes.

1. Introduction

The purpose of the present paper is to derive the Martin representation of nonnegative functions which are α-harmonic on Lipschitz domains. In general any nonnegative function α-harmonic in a bounded domain decomposes into regular part and singular part [9]. Each part admits its Martin representation. The regular part is given by integrating the function outside the domain against the α-harmonic measures. In [3] we have seen that for Lipschitz domains the α-harmonic measures have jointly continuous densities with respect to the Lebesgue measure. In the present paper, by using the boundary Harnack principle proved in [3], we shall show that the singular part is given by integrating a family of singular α-harmonic functions which are parameterized by the points at the boundary. Our restriction to Lipschitz domains simplifies the argument and yields an explicit definition of the family in terms of the Green function.

Our development is a standard one (cf. [1]). Nevertheless most of the arguments are substantially modified in comparison with its classical counterpart. We use this opportunity to illustrate the theory of Martin representation by a straightforward construction. Martin representation in the case of arbitrary bounded domains is the subject of [9]. Reader interested in the Martin representation in a general setting of Markov processes is referred to [7].

We first review the notation and a few results on α-harmonic functions following [3]. For the rest of the paper, let $\alpha \in (0,2)$ and $d \geq 2$. We denote by (X_t, P^x) the standard rotation invariant α-stable Lévy process (i.e. homogeneous and with independent increments) in $\mathbf{R}^d$ with the characteristic

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