HIROSHIMA MATH. J.
29 (1999), 579–590

Enumerating embeddings of *n*-manifolds into complex projective *n*-space

Dedicated to Professor Fuichi Uchida on his 60th birthday

Tsutomu YASUI

(Received November 4, 1998)

ABSTRACT. Let $f : M \to N$ be an embedding between differentiable manifolds and set $\pi_1(N^M, \mathrm{Emb}(M,N), f) = [M \subset N]_f$, where $\mathrm{Emb}(M,N)$ denotes the space of embeddings of M to N. Then it is known that there is a $\pi_1(N^M, f)$-action on $[M \subset N]_f$ such that $[M \subset N]_f / \pi_1(N^M, f)$ is equivalent to the set $[M \subset N]_{[f]}$ of isotopy classes of embeddings homotopic to f. In this paper, we will study the set $[M^n \subset CP^n]_f$ for an n-manifold M^n. Further we will determine the sets $[RP^n \subset CP^n]_{[f]}$ and $[CP^n \subset CP^{2n}]_{[f]}$.

1. Introduction and statement of results

Throughout this paper, n-manifolds mean n-dimensional connected differentiable manifolds without boundary and embeddings stand for differentiable embeddings of compact manifolds to manifolds. For any map $f : M \to N$, we denote by $[M \subset N]_{[f]}$ the set of isotopy classes of embeddings homotopic to f. A. Haefliger's existence theorem [3] implies that for any compact n-manifold M^n and any map $f : M^n \to CP^n$ $(n > 2)$, there exists an embedding homotopic to f. Henceforth we would like to determine the set $[M^n \subset CP^n]_{[f]}$.

Set $\pi_1(N^M, \mathrm{Emb}(M,N), f) = [M \subset N]_f$, where $\mathrm{Emb}(M,N)$ denotes the space of embeddings of M to N. Then it is known (cf. [2], [7], [8], [12]) that there is a $\pi_1(N^M, f)$-action on $[M \subset N]_f$ such that

$$(1.1) \qquad [M \subset N]_f / \pi_1(N^M, f) = [M \subset N]_{[f]}.$$

In this paper, we will study the set $[M^n \subset CP^n]_f$ for an n-manifold M^n and a map $f : M^n \to CP^n$. Furthermore we will determine the isotopy sets of embeddings $[RP^n \subset CP^n]_{[f]}$ and $[CP^n \subset CP^{2n}]_{[f]}$.

The integral cohomology of CP^n is given by

$$H^*(CP^n; Z) = Z[z]/(z^{n+1})(\deg z = 2).$$

1991 *Mathematics Subject Classification.* Primary 57R52; Secondary 57R40
Key words and phrases. embedding, isotopy, 2-fold symmetric product