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On the Gysin isomorphism of rigid cohomology

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ABSTRACT. We prove a comparison theorem of logarithmic Monsky-Washnitzer cohomology and rigid cohomology with overconvergent coefficients. Using this comparison theorem, we construct the Gysin isomorphism in rigid cohomology with overconvergent coefficients on small pairs of affine smooth varieties of positive characteristic. The Gysin isomorphism under the assumption "small" is sufficient to apply it to the finiteness problem of rigid cohomology with coefficients. We prove the finiteness theorem, Poincaré duality and Künneth formula of rigid cohomology for unit-root overconvergent F-isocrystals by our previous result of finite local monodromy theorem for them.

1. Introduction

The rigid cohomology with coefficient of overconvergent isocrystals, which was introduced by P. Berthelot, is a good candidate of the p-adic cohomology theory of varieties of positive characteristic p. If the rigid cohomology is a good cohomology, then it must have several expected properties, the finiteness, Poincaré duality, Künneth formula and so on. In [6] and [7] Berthelot proved the finiteness, Poincaré duality and Künneth formula of the rigid cohomology with the constant coefficient. In his proof the Gysin isomorphism played an important role.

In this article we construct the Gysin isomorphism of the rigid cohomology of overconvergent isocrystals on sufficiently small affine smooth varieties. For overconvergent F-isocrystals, this Gysin isomorphism commutes with Frobenius structures. We apply it to the finiteness, Poincaré duality and Künneth formula of the rigid cohomology of overconvergent unit-root F-isocrystals.

Let us explain the method of the construction of the Gysin isomorphism. First we introduce a logarithmic Monsky-Washnitzer cohomology and prove the comparison theorem with overconvergent coefficients between the logarithmic Monsky-Washnitzer cohomology and the rigid cohomology for an affine smooth variety with normal crossing divisor over a spectrum of field of positive characteristic. This comparison theorem is a p-adic analogue of A. Grothendieck and P. Deligne's comparison theorem of the logarithmic

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