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On hypercoverings

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ABSTRACT. In the paper [3], a new notion of hypercovering was introduced to compute the higher cohomology groups of hypercoverings. This hypercovering has no degeneracy maps, and hence it is not a simplicial scheme.

In this paper, we study the basic properties of these hypercoverings. It has the notion of cosquelton like usual hypercoverings, and one can construct the hypercovering inductively using cosqueltons.

We construct intermediate objects corresponding to each simplicial ordered set. For example, the n-th cosquelton corresponds to $n-1$ sphere, and the n-th hypercovering corresponds to n-simplex. To each simplicial map corresponds a morphism contravariantly. Our main theorem says that this morphism is always a covering map.

1. Ordered system

DEFINITION 1.1. Let $\mathcal{N}$ be the category of *finite strictly ordered sets*. Namely, its objects are finite sets $\{\phi, \{1\}, \{1,2\}, \{1,2,3\}, \dots\}$, and morphisms are strictly increasing, say $f(i) < f(j)$ when $i < j$ (hence always injective). We denote the finite ordered set $\{1,2,\dots,n\}$ simply by $\mathbf{n}$ when confusion is unlikely. With this notation, the objects of $\mathcal{N}$ are $\{\mathbf{0},\mathbf{1},\mathbf{2},\mathbf{3},\dots\}$.

Define the morphism $\partial_i : \mathbf{n} \to \mathbf{n+1}$ for $i = 1,2,\dots,n+1$ by

$$\partial_i(j) = \begin{cases} j & (i > j) \\ j+1 & (i \le j) \end{cases}.$$

The truncated ordered set $\mathcal{N}_{[n]}$ is the full subcategory of $\mathcal{N}$ with objects $\{\mathbf{0},\mathbf{1},\mathbf{2},\mathbf{3},\dots,\mathbf{n}\}$.

REMARK 1.1.1. It is easy to check the following identity.

$$\partial_i \circ \partial_j = \begin{cases} \partial_j \circ \partial_{i-1} & (i > j) \\ \partial_{j+1} \circ \partial_i & (i \le j) \end{cases} \tag{1}$$

Any morphism $f : \mathbf{n} \to \mathbf{m}$ can be written as a composition of ∂_i's. In fact, when the set $\{i_1, \dots, i_{m-n}\}$ is the complement of the set $\{f(1), f(2), \dots, f(n)\}$

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