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Homotopy classification of higher homotopy commutative loop spaces with finitely generated cohomology

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ABSTRACT. Suppose X is a simply connected mod p loop space such that the mod p cohomology $H^*(X)$ is finitely generated as an algebra. We show that if X is a C_n-space in the sense of Williams then X is the total space of a C_n-fibration over a finite C_n-space. By using this result, we can reduce problems about C_n-spaces with finitely generated cohomology to the case of finite C_n-spaces. In particular, we give classification theorems for C_2-spaces and C_p-spaces with finitely generated cohomology.

1. Introduction

Loop space plays an important role in homotopy theory of Lie groups, and it has been investigated from several points of view (cf. [15], [19], [25], [26]). It is convenient to consider the loop space at a prime by using completion theory due to Bousfield-Kan [3]. Let p be a prime. A loop space which is completed at p is called a mod p loop space. Throughout the paper, homotopy equivalence means mod p homotopy equivalence and cohomology is mod p cohomology unless otherwise specified.

Dwyer-Wilkerson [11] defined the p-compact group and studied its properties. A loop space X is said to be a p-compact group if the classifying space BX is p-completed and the mod p cohomology $H^*(X)$ is finite dimensional. Besides compact Lie groups, other useful examples of p-compact groups are known. In fact, the p-completion of an odd dimensional sphere S^{2n-1} is a p-compact group if $n|(p-1)$. In recent years, many theorems have been proved about p-compact groups (cf. [11], [26]), and those theorems suggest that p-compact groups have similar properties to those of Lie groups.

In this paper, we consider loop spaces which need not be finite, but whose mod p cohomology rings are finitely generated. Recently, Broto and Crespo [4], [8] gave remarkable results for H-spaces with finitely generated cohomology. It follows from their results that a mod p loop space with

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