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An axiomatic approach to the cut-off phenomenon for random walks on large distance-regular graphs

To the Memory of Dr. Hitoshi Mizumachi

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ABSTRACT. The cut-off phenomenon is a sort of critical phenomenon which one often observes in the process of convergence to equilibrium for various Markov chains including card shuffling and diffusion of sparse gases. This article aims at developing an axiomatic approach to this phenomenon on a nice class of distance-regular graphs. Following the formulation through large volume limits, we present a rigorous criterion for the cut-off phenomenon in terms of spectral data of the adjacency matrix of the graph.

1. Introduction

The *cut-off phenomenon* (abbreviated to COP) is widely observed in the process of convergence to equilibrium for Markov chains. It is a critical phenomenon owing to the huge cardinality of the state space of the chain and is well understood through a large volume limit of the system. Initiated by P. Diaconis, the study of this phenomenon has now grown to enjoy considerable literature. Let us consider a Markov chain on finite state space X with transition probability matrix P and invariant probability π. We assume the convergence to equilibrium of the chain:

$$(P^k)_{x,y} \to \pi(y) \qquad \text{as } k \to \infty \qquad \text{for } \forall x, y \in X,$$

which is in fact assured under mild conditions. The total variation distance

$$\|(P^k)_{x,\cdot} - \pi\| = \frac{1}{2}\sum_{y \in X} |(P^k)_{x,y} - \pi(y)| \tag{1}$$

will describe the convergence more quantitatively. In this article, we treat Markov chains enjoying some spatial symmetry, which then implies that the

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