HIROSHIMA MATH. J.
30 (2000), 215–219

Liftings of Pettis integrable functions

K. MUSIAŁ and N. D. MACHERAS

(Received December 7, 1998)
(Revised May 27, 1999)

ABSTRACT. It is proved, that if the lifting of a bounded Pettis integrable function is appropriately measurable, then it is also Pettis integrable.

Introduction

Throughout this paper (Ω, Σ, μ) is a finite complete measure space, ρ is a lifting on $L_\infty(\mu)$, X denotes an arbitrary Banach space and X_c^{**} is the set of all $x^{**} \in X^{**}$ which are weak*-cluster points of bounded countable subsets of X.

If $f : \Omega \to X$ is a weakly measurable and scalarly bounded (or $f : \Omega \to X^{**}$ is weak*-measurable and weak*-bounded), then $\rho_1(f) : \Omega \to X^{**}$ is the unique function satisfying for each $x^* \in X^*$ and each $\omega \in \Omega$ the equality $\langle \rho_1(f)(\omega), x^* \rangle = \rho(x^* f)(\omega)$ (cf [2], VI. 4). If $f : \Omega \to X^*$ is a weak*-measurable and weak*-bounded, then $\rho_0(f) : \Omega \to X^*$ is the unique function satisfying for each $x \in X$ and each $\omega \in \Omega$ the equality $\langle \rho_0(f)(\omega), x \rangle = \rho(xf)(\omega)$.

It is known (cf [6]), that $\rho_1(f) : \Omega \to X^{**}$ and $\rho_0(f) : \Omega \to X^*$ are weak*-Borel measurable and the measures $\xi_0 := \mu\rho_0(f)^{-1}$ and $\xi_1 := \mu\rho_1(f)^{-1}$ are Radon measures on the completions Ξ_f^0 and Ξ_f^1 of the σ-algebras of weak*-Borel subsets of X^* and X^{**} respectively. If $E \in \Sigma$ and $f : \Omega \to X$ is Pettis integrable, then $\nu_f : \Sigma \to X$ is given by $\nu_f(E) := \int_E f \, d\mu$. The space of all X-valued μ-Pettis integrable functions is denoted by $P(\mu, X)$ (weakly equivalent functions are identified).

It is an open problem, whether the functions $\rho_0(f)$ and $\rho_1(f)$ are always Pettis integrable if f is bounded and Pettis integrable. Talagrand presented a few sufficient conditions in [6]. In particular, the RS-property is sufficient for the Pettis integrability of $\rho_0(f)$ for a bounded f but as he noticed in 7-3-16 of [6], the Pettis integrability of $\rho_0(f)$ need not imply the RS-property of f. In 1996 Rybakov published a paper [5] concerning the Pettis integrability of $\rho_1(f)$

2000 *Mathematics Subject Classification*. Primary: 46G10; Secondary: 28B05, 28A15.
Key words and phrases. Pettis integral, lifting.
The research of the first author was supported by KBN Grant No 2 P03A 025 11.