HIROSHIMA MATH. J.
30 (2000), 499–512

A white noise approach to an evolutional equation in biology

Hisayuki YONEZAWA

(Received November 8, 1999)

ABSTRACT. In this paper we shall discuss a white noise differential equation that comes from some biological phenomena. Having applied the so-called S-transform to the white noise functionals the given equation turns into an equation for a U-functional. There the advantage of the white noise calculus is heavily used. Thus the solution is obtained in an explicit form in terms of white noise, and we see that the solution is a generalized functional which is in the space of Cochran-Kuo-Sengupta. Some characteristic properties of the solution are shown, for instance, positivity of the solution. Further, its mean lies in between 0 and 1. The expression of the solution shows the most significant property that there is an *asymmetry* in time for the phenomenon in question.

1. Introduction

A biological organism is composed of one cell or many cells. The surface of a cell is covered with a plasma membrane and the membrane is the border between the inside and the outside of a cell. Disproportion of sodium, potassium, calcium and chlorine ions exist between the inside and the outside of a cell and this fact brings about the difference in electric potential. Ion channels are macromolecules that open and close in a random fashion on membrane and play the role of gatekeepers which control the flux of their ions coming in and out of the cell.

F. Oosawa et al. [10] has introduced a differential equation which is proposed to describe the probabilistic behavior of ion channels in a fluctuating electric field. They claim that the open—close fluctuation in an assembly of channels has an asymmetry with respect to time reversal. In their theory, the probability which is the ratio of the number of channels in open state to the total number of channels is denoted by $p(t)$ and it is given by the solution to the equation.

$$\frac{dp(t)}{dt} = -k_{+o}\exp\{-\beta E(t)\}p(t) + k_{-o}\exp\{+\beta E(t)\}(1-p(t)), \tag{1.1}$$

where $k_{+o} > 0$, $k_{-o} > 0$, $\beta = (1/2)\delta\mu/kT$ ($\delta\mu$: the free energy difference between

2000 *Mathematics Subject Classification.* 60H40.
Key words and phrases. White noise, Infinite dimensional Calculus.