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Chemotactic collapse in a parabolic system of mathematical biology

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ABSTRACT. In 1970, Keller and Segel proposed a parabolic system describing the chemotactic feature of cellular slime molds and recently, several mathematical works have been devoted to it. In the present paper, we study its blowup mechamism and prove the following. First, chemotactic collapse occurs at each isolated blowup point. Next, any blowup point is isolated, provided that the Lyapunov function is bounded from below. Finally, only the origin can be a blowup point of radially symmetric solutions.

1. Introduction

A system of parabolic partial differential equations of mathematical biology is attracting interest. It was proposed by Nanjundiah [22] in 1973, as a simplified model of the Keller and Segel system [16] describing a chemotactic feature, the aggregation of some organisms (cellular slime molds) sensitive to gradient of a chemical substance. Precisely, with $u(x,t)$ and $v(x,t)$ standing for the density of the organism and the concentration of the chemical substance at the position $x \in \Omega$ and the time $t \in (0,T)$, respectively, it is given as

$$\text{(KS)} \qquad \begin{cases} \dfrac{\partial u}{\partial t} = \nabla \cdot (\nabla u - \chi u \nabla v) & \text{in } \Omega \times (0,T) \\ \tau \dfrac{\partial v}{\partial t} = \Delta v - \gamma v + \alpha u & \text{in } \Omega \times (0,T) \\ \dfrac{\partial u}{\partial n} = \dfrac{\partial v}{\partial n} = 0 & \text{on } \partial\Omega \times (0,T) \\ u(\cdot,0) = u_0, \qquad v(\cdot,0) = v_0 & \text{on } \Omega, \end{cases}$$

where

(A1) τ, α, γ and χ are positive constants
(A2) Ω is a bounded domain in $\mathbf{R}^2$ with smooth boundary $\partial\Omega$
(A3) n denotes the unit outer normal vector

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