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Existence and nonexistence of positive radial entire solutions of second order quasilinear elliptic systems

Tomomitsu TERAMOTO

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ABSTRACT. This paper treats the second order quasilinear elliptic system of the form $\Delta_p u = H(|x|)v^\alpha, \Delta_q v = K(|x|)u^\beta$ in $\mathbf{R}^N$ with nonnegative functions H, K. Sufficient conditions will be given to have positive radial entire solutions and to have no nonnegative nontrivial radial entire solutions under some restriction on p, q, α and β. When H and K behave like positive constant multiples of $|x|^\nu$, $\nu \in \mathbf{R}$, we can completely characterize the existence property of positive radial entire solutions.

1. Introduction and statement of results

This paper is concerned with second order quasilinear elliptic system of the form

$$(1) \qquad \begin{cases} \Delta_p u \equiv \operatorname{div}(|Du|^{p-2}Du) = H(|x|)v^\alpha \\ \Delta_q v \equiv \operatorname{div}(|Dv|^{q-2}Dv) = K(|x|)u^\beta \end{cases} \quad \text{in } \mathbf{R}^N,$$

where $N \geq 1$, $p > 1$, $q > 1$, α and β are positive constants satisfying $\alpha\beta > (p-1)(q-1)$, and $H, K : [0, \infty) \to [0, \infty)$ are continuous. An *entire solution* of (1) is defined to be a function $(u, v) \in C^1(\mathbf{R}^N) \times C^1(\mathbf{R}^N)$ such that $|Du|^{p-2}Du, |Dv|^{q-2}Dv \in C^1(\mathbf{R}^N)$ and satisfies (1) at every $x \in \mathbf{R}^N$. Such a solution of (1) is said to be radial if it depends only on $|x|$.

The problem of existence and nonexistence of positive radial entire solutions of *scalar* equations has been investigated by many authors under various situations. To illustrate some of typical known results let us consider the equation

$$(2) \qquad \Delta_p u = H(x)u^\sigma \quad \text{in } \mathbf{R}^N,$$

where $p > 1$, $\sigma > p - 1$, and H is a nonnegative continuous function in $\mathbf{R}^N$. The existence and nonexistence results of positive (radial) entire solutions of (2) may be described roughly as follows:

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