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On non-singular stable maps of 3-manifolds with boundary into the plane

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ABSTRACT. Let M be a compact connected orientable 3-manifold with non-empty boundary and $f : M \to \mathbf{R}^2$ a stable map. In this paper we study the existence of an immersion or embedding lift of f to $\mathbf{R}^n$ $(n \geq 3)$ with respect to the standard projection $\mathbf{R}^n \to \mathbf{R}^2$. We also characterize the orientable 3-dimensional handlebody in terms of stable maps which have only a restricted class of singularities. Moreover, by using the concept of an embedding lift of a certain map of a 2-dimensional polyhedron into $\mathbf{R}^2$, we give a characterization of S^3.

1. Introduction

Let M be a smooth manifold, $f : M \to \mathbf{R}^m$ a smooth map and $\pi : \mathbf{R}^n \to \mathbf{R}^m$ $(n > m)$ a standard projection. Then we ask if there exists an immersion or embedding $g : M \to \mathbf{R}^n$ which satisfies $f = \pi \circ g$. Such a map g is called an *immersion* or *embedding lift* of f.

In this paper, M will be a compact connected orientable 3-manifold with non-empty boundary, of class C^∞. Let $f : M \to \mathbf{R}^2$ be a stable map. We ask if there exists an immersion or embedding lift of f to $\mathbf{R}^n$ $(n \geq 3)$ with respect to the standard projection $\pi : \mathbf{R}^n \to \mathbf{R}^2$, $(x_1, x_2, \ldots, x_n) \mapsto (x_1, x_2)$. A point x in M is called a *singularity* if $\operatorname{rank} df_x < 2$. $S(f)$ denotes the set of singularities of f. Our main result is the following theorem.

THEOREM 1. *Let M be a compact connected orientable 3-manifold with non-empty boundary and $f : M \to \mathbf{R}^2$ a stable map. We consider the condition* (I)*: For any $r \in \mathbf{R}^2$, $f^{-1}(r)$ is either empty or homeomorphic to a finite disjoint union of closed intervals and points. Then the following two conditions are equivalent.*

(a) *f has an immersion lift to $\mathbf{R}^3$.*

(b) *$S(f) = \emptyset$ and f satisfies the condition* (I).

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