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Spatial homogenization and internal layers in a reaction-diffusion system

Dedicated to Professor Jack Hale for his 70th birthday

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ABSTRACT. For a system of reaction-diffusion equations of activator-inhibitor type, we show that solutions undergo at least three stages of dynamical behaviour when the activator diffuses slowly and reacts fast, and the inhibitor diffuses fast. In the first stage, the inhibitor quickly decays to its spatial average (spatial homogenization of the inhibitor). In the second stage, the activator develops internal layers (formation of internal layers). In the third stage, the layers move according to a certain motion law (motion of interfaces) which is described by a system of ordinary differential equations on finite time intervals. Asymptotic behaviour of the solutions of the interface equation is also analyzed. To describe the behaviour of the solutions of the reaction-diffusion equations after the last interface equation becomes powerless, another type of interface equation is proposed.

1. Introduction

The reaction-diffusion system

$$u_t = d_1 \Delta u + f(u, v), \qquad v_t = d_2 \Delta v + rg(u, v)$$

has been employed to model propagation phenomena of chemical waves in excitable media [6], and to describe pattern formation in an activator-inhibitor model [10]. In this system, $d_1 > 0$, $d_2 > 0$ are diffusion rates of u and v, and $r > 0$ measures the ratio of the reaction rates of u and v. Depending upon the relative magnitude among d_1, d_2 and r, it has been found by many authors that the system above, despite its simplicity, is capable of producing various spatio-temporal patterns such as propagating fronts and localized spatial structures [14]. These studies indicate that various patterns observed in reacting and diffusing systems are produced by the interaction between local reaction kinetics and global diffusion effects. It is therefore important to mathematically study

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