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The harmonic product of $\delta(x_1, \ldots, x_n)$ and $\delta(x_1)$ and two combinatorial identities

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ABSTRACT. In the framework of nonstandard analysis, Bang-He Li and the author defined the product of any two distributions on R^n via their harmonic representations. The product of $\delta(x_1, \ldots, x_n)$ and $\delta(x_1)$ was calculated by Kuribayashi and the author in [LK]. In this paper, the result of [LK] is improved to

$$\delta(x_1, \ldots, x_n) \circ \delta(x_1) = \frac{1}{2\pi\rho}\delta(x_1, \ldots, x_n) \mod \text{infinitesimals}$$

where ρ is a positive infinitesimal. Moreover two combinatorial identities are obtained as byproducts.

Thirty years ago, Bremermann and Durand [BD] defined the products of distributions with one variable by using analytic representations. It was shown by Itano [I1][I2] and further by Bang-He Li and the author [LL1] that this multiplication is very broad, i.e. if the product of two distributions exists for several other multiplications, then the same product is obtained for this multiplication. So a problem that interested people was "What is a generalization of this multiplication to distributions with several variables?".

Itano [I1] showed by an example that for distributions with more than one variables, analytic representation can not offer well-defined multiplication. Bang-He Li and the author [LL2] at last found that a suitable generalization of this multiplication is the one via harmonic representations; For one variable, harmonic and analytic representations are essentially the same. Bang-He Li [L] adapted the multiplication of Bremermann and Durand into the framework of nonstandard analysis. Its generalization to multiple variables via harmonic representations [LL2] was also written in this framework. The merit to use nonstandard analysis is that one needs not to worry about the existence of products anymore, and when taking the finite part (if exists), we return to some kind of standard product.

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