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Tilings from non-Pisot unimodular matrices

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Abstract. Using the unimodular Pisot substitution of the free monoid on d letters, the existence of graph-directed self-similar sets $\{X_i\}_{i=1,2,\dots,d}$ satisfying the set equation (0.0.1) with the positive measure on the A-invariant contracting plane P is well-known, where A is the incidence matrix of the substitution. Moreover, under some conditions, the set $\{X_i\}_{i=1,2,\dots,d}$ is the prototile of the quasi-periodic tiling of P (see Figure 1). In this paper, even in the case of non-Pisot matrix A, the generating method of graph-directed self-similar sets and quasi-periodic tilings is proposed under the "blocking condition".

0. Introduction

The following fact is well-known: using the unimodular Pisot substitution σ of the free monoid on d letters, we obtain the prototiles $\{X_i\}_{i=1,2,\dots,d}$ with fractal boundary of the A-invariant contracting plane P, satisfying the set equation:

$$A^{-1}X_i = \bigcup_{j=1}^{l_i}(\boldsymbol{v}_j^{(i)} + X_j) \qquad \text{(non-overlapping)} \tag{0.0.1}$$

where the transformation A is the incidence matrix of the substitution σ and vectors $\boldsymbol{v}_j^{(i)} \in P$, $1 \le j \le l_i$ are some translations. Moreover, under the super coincidence condition in [14], we see that the prototiles $\{X_i\}_{i=1,2,\dots,d}$ give us a graph directed self-similar tiling of P (see Figure 1). The prototiles from the substitution have been studied first by Rauzy in [20]. Since Rauzy (see Figure 1), several properties of prototiles have been studied by many authors. For example, basic properties of $\{X_i\}_{i=1,2,\dots,d}$ have been studied in [16], [4], [10], [21] and [2], the estimation of the Hausdorff dimention of ∂X_i in [10], topological properties of X_i in [22], [1], the relation with the Markov partition generated by $\{X_i\}_{i=1,2,\dots,d}$ in [4], [18], the relation with the algebraic β-expansion in [15], [14], Diophantine approximation in [13], quasi-periodic tiling in [14], [17], etc. In fact, we know that to study the structure of $\{X_i\}_{i=1,2,\dots,d}$ is useful and

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