HIROSHIMA MATH. J.
36 (2006), 221–234

Liouville theorems of stable F-harmonic maps for compact convex hypersurfaces

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(Received March 28, 2005)
(Revised July 21, 2005)

ABSTRACT. Let M^n be a compact convex hypersurface in $\mathbf{R}^{n+1}$. In this paper, we proved firstly that if the principal curvatures λ_i of M^n satisfy $0 < \lambda_1 \leq \cdots \leq \lambda_n$ and $\lambda_n < \sum_{j=1}^{n-1} \lambda_j$, then there exist no nonconstant stable F-harmonic map between M and a compact Riemannian manifold when (1.2) or (1.3) holds (Theorem 1). This is a generalization or unification of the corresponding results for several varieties of harmonic map. Then, when the target manifold is δ-pinched, using a new estimate method, we obtain the Liouville-type theorem (Theorem 2) for stable F-harmonic map, which improves the results of M. Ara in [2].

1. Introduction

The instability for harmonic map (as well as p-harmonic map), from or into standard unit sphere S^n in Euclidean space $\mathbf{R}^{n+1}$, is well-known. For example, there exists no nonconstant stable harmonic (or p-harmonic) map either from S^n to any Riemannian manifold [12] (or [11]), or from any compact Riemannian manifold to S^n [6] (or [3]). In this paper, for a smooth function $F : [0, \infty) \to [0, \infty)$ such that $F'(t) > 0$ on $t \in (0, \infty)$, we concern with the instability of F-harmonic maps which is the generalization and union of the harmonic, p-harmonic or exponentially harmonic maps, introduced by M. Ara in [2].

M. Ara [1] proved that every stable F-harmonic map $u : M \to S^n$ is constant, provided that

$$\int_M |du|^2 \left\{ |du|^2 F''\left(\frac{|du|^2}{2}\right) + (2-n)F'\left(\frac{|du|^2}{2}\right) \right\} *1 < 0. \tag{1.1}$$

In contrast with this, as far as I know there is few result when the source manifold is S^n. In this paper, however, we can prove the following instability

2000 *Mathematics Subject Classification*. 58E20.

Key words and phrases. F-harmonic maps, instability, compact convex hypersurfaces, δ-pinched manifolds.

Project supported by Chinese Tianyuan Mathematics Fund (A0324662) and National Natural Science Foundation of China (10571129).