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Metacyclic groups of automorphisms of compact Riemann surfaces

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Abstract. Let $G_s^0(n) = \langle a, b : a^n = 1 = b^2, b^{-1}ab = a^s \rangle$ and for n even $G_s^1(n) = \langle a, b : a^n = 1, a^{n/2} = b^2, b^{-1}ab = a^s \rangle$. In this paper we compute the minimum genus $g^* \geq 2$ of a compact Riemann surface that admits a metacyclic group $G_s^0(n)$ or $G_s^1(n)$ of biholomorphic homeomorphisms.

1. Introduction

It is known that for any compact Riemann surface of genus ≥ 2 the group of biholomorphic homeomorphisms, which we call automorphisms, is finite [9, p. 66] and that every finite group can be so realized [2 and 3]. Therefore, the following problem arises: Given a finite group G what is the minimum genus $g^* \geq 2$ of a compact Riemann surface that admits G as a group of automorphisms?

We solve this problem for G metacyclic group that belongs to two special classes of extensions of a cyclic group by an involution namely the classes

$$G_s^0(n) = \langle a, b : a^n = 1 = b^2, b^{-1}ab = a^s \rangle \qquad \text{and}$$

$$G_s^1(n) = \langle a, b : a^n = 1, a^{n/2} = b^2, b^{-1}ab = a^s \rangle \qquad \text{with } n \text{ even.}$$

The solution of this problem for these two classes is given in Theorems 3.3 and 4.2 respectively.

The same problem has been solved for cyclic and abelian groups in [4] and [8] respectively. This paper treats the non-abelian case for the first time.

2. The Fuchsian group approach

We shall approach the problem using Fuchsian groups. All the details and the proofs of the following well-known facts can be found in [7] and [6], see also [5].

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