

Extendibility and stable extendibility of vector bundles over real projective spaces

Teiichi KOBAYASHI and Kazushi KOMATSU

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ABSTRACT. The purpose of this paper is to study the extendibility and the stable extendibility of vector bundles of real projective spaces and those of their complexifications. We determine the dimension m for which the complexification of the tangent bundle of the n -dimensional real projective space RP^n is extendible to RP^m for $n = 6$ or $n > 7$, and determine the dimension n for which the square of the tangent bundle of RP^n or its complexification is extendible to RP^m for every $m > n$.

1. Introduction

Let X be a space and A be its subspace. A t -dimensional F -vector bundle ζ over A is called extendible (respectively stably extendible) to X , if there is a t -dimensional F -vector bundle over X whose restriction to A is equivalent (respectively stably equivalent) to ζ as F -vector bundles, where F is the real number field R , the complex number field C or the quaternion number field H (cf. [8] and [3]). Let R^n be the n -dimensional Euclidean space, RP^n be the n -dimensional real projective space and $\tau(RP^n)$ be the tangent bundle of RP^n .

First, we study the question: Determine the dimension m with $m > n$ for which a vector bundle over RP^n is extendible to RP^m . We have obtained the complete answer for the tangent bundle $\tau(RP^n)$ in [4, Theorem 6.6].

For an R -vector bundle and a C -vector bundle over RP^n we have

THEOREM 1. *Let ζ be a t -dimensional R -vector bundle over RP^n . If $n < t$, ζ is extendible to RP^m for every m with $n < m \leq t$.*

THEOREM 2. *Let ζ be a t -dimensional C -vector bundle over RP^n . If $n < 2t + 1$, ζ is extendible to RP^m for every m with $n < m \leq 2t + 1$.*

For the complexification of the tangent bundle $\tau(RP^n)$, we have

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