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A splitting theorem for rank two vector bundles on projective spaces in positive characteristic

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ABSTRACT. We shall prove the following splitting theorem for rank two vector bundles E on the n-dimensional projective space $\mathbf{P}^n$ $(n \geq 4)$ in positive characteristic. Let P be a 4- or 5-dimensional projective linear subspace of $\mathbf{P}^n$ and $\bar{E} = E|P$ the restriction of E to P. Then E splits into line bundles if and only if the first cohomology of the sheaf of endomorphisms of $\bar{E}$ vanishes.

0. Introduction

Let E be a rank two vector bundle on the n-dimensional projective space $\mathbf{P}_k^n$ $(n \geq 4)$ defined over an algebraically closed field k.

In [4], H. Sumihiro showed the following theorem in the case of char $k = 0$.

THEOREM 0.1. *Let P be a* 4- *or* 5-*dimensional projective linear subspace of $\mathbf{P}_k^n$ and $\bar{E} = E|P$ the restriction of E to P. Then E splits into line bundles if and only if $H^1(P, \mathscr{E}nd(\bar{E})) = 0$.*

The aim of this article is to prove that this theorem holds also true in char $k = p > 0$. The proof is almost the same as the one for char $k = 0$, namely, it is obtained by studying some geometric structures of the Hilbert scheme of $\mathbf{P}_k^n$ at determinantal subvarieties. In char $k = p > 0$, however, since we cannot use the Kodaira vanishing theorem and the Le-Potier vanishing theorem (cf. [1], [3]), we have to observe some vanishings of cohomologies appearing in [4] carefully.

1. Preliminaries

We first recall the definition and some properties of determinantal varieties associated to rank two bundles (cf. [4]).

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