HIROSHIMA MATH. J.
31 (2001), 461–465

A note on nonremovable cusp singularities

Dedicated to Professors Takuo Fukuda and Shuzo Izumi on their 60-th birthdays

Kazuhiro SAKUMA

(Received April 9, 2001)
(Revised May 21, 2001)

ABSTRACT. Let M^4 be a closed oriented 4-manifold such that $\mathrm{rank}_{\mathbf{Z}_2} H_2(M^4; \mathbf{Z}_2) = 1$. Then we prove that every stable map $f : M^4 \to \mathbf{R}^3$ always has cusp singularities.

1. Introduction

Is the Thom polynomial the only obstruction to removing the corresponding singularities of a given smooth map? This is one of the important questions in Global Singularity Theory and several affirmative answers had been given mainly on the elimination of 0-dimensional singularities (see [9, 8]). In [5, Theorem 4] and [6, Theorem 4.5] Saeki discovered a negative case that every stable map of a closed 4-dimensional manifold M^4 with $H_*(M^4; \mathbf{Z}) \cong H_*(\mathbf{C}P^2; \mathbf{Z})$ into an orientable 3-manifold has necessarily nonempty one dimensional cusp singularities. This means that the cusp singularities cannot be removed, although the Thom polynomial vanishes.

Analyzing his proof in [5], it seems that this phenomenon is caused by a Rohlin type theorem peculiar to 4-dimensional differential topology and essentially based on the condition that $H_2(M^4; \mathbf{Z}) \cong \mathbf{Z}$. The condition that $H_1(M^4; \mathbf{Z}) = 0$ is also technically needed to use the author's mod 4 formula in [10].

The purpose of this paper is to extend Saeki's theorem for a wider class of 4-manifolds at least without the assumption that $H_1(M^4; \mathbf{Z}) = 0$. Actually, we prove that the same statement holds for a closed orientable 4-manifold M^4 with $\mathrm{rank}_{\mathbf{Z}_2} H_2(M^4; \mathbf{Z}_2) = 1$. Our proof heavily depends on Yamada's result in [13].

2000 *Mathematics Subject Classification*. 57R45, 57R42.
Key words and phrases. Thom polynomial, stable map, cusp singularity, 4-manifold.
Partially supported by Grant-in-Aid for Scientific Research (C), Japan Society for the Promotion of Science (No. 11640096) and Grant for the Encouragement of Young Researchers, Kinki University (G008).