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On the Cowling-Price theorem for $SU(1,1)$

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Abstract. M. G. Cowling and J. F. Price showed a kind of uncertainty principle on Fourier analysis. If v and w grow very rapidly then the finiteness of $\|vf\|_p$ and $\|w\hat{f}\|_q$ implies that $f = 0$, where $\hat{f}$ denotes the Fourier transform of f. We give an analogue of this theorem for $SU(1,1)$.

1. Introduction

The Hardy theorem asserts that if a measurable function f on $\mathbf{R}$ satisfies $|f(x)| \le Ce^{-ax^2}$ and $|\hat{f}(y)| \le Ce^{-by^2}$ and $ab > \frac{1}{4}$ then $f = 0$ (a.e.). Here we use the Fourier transform defined by $\hat{f}(y) = (1/\sqrt{2\pi})\int_{-\infty}^{\infty} f(x)e^{\sqrt{-1}xy}\,dx$. M. G. Cowling and J. F. Price [4] generalized the Hardy theorem as follows: Suppose that $1 \le p, q \le \infty$ and one of them is finite. If a measurable function f on $\mathbf{R}$ satisfies $\|\exp\{ax^2\}f(x)\|_{L^p(\mathbf{R})} < \infty$ and $\|\exp\{by^2\}\hat{f}(y)\|_{L^q(\mathbf{R})} < \infty$ and $ab \ge 1/4$ then $f = 0$ (a.e.). The case where $p = q = \infty$ and $ab > 1/4$ is covered by the Hardy theorem. S. C. Bagchi and S. K. Ray [1] showed that if $ab > 1/4$, then the Hardy theorem is equivalent to the Cowling-Price theorem.

A. Sitaram and M. Sundari [14] obtained the Hardy theorem in the case of noncompact semisimple Lie groups with one conjugacy class of Cartan subgroups, $SL(2,\mathbf{R})$ and Riemannian symmetric spaces of the noncompact type. Recently J. Sengupta [12] and M. Ebata et al. [6] obtained the Hardy theorem for all Lie groups of Harish-Chandra class and all connected semisimple Lie groups with finite center respectively. Also, M. Cowling, A. Sitaram and M. Sundari [5] gave another simple proof of the Hardy theorem for connected real semisimple Lie groups with finite center. On the other hand, S. C. Bagchi and S. K. Ray [1] obtained the Cowling-Price theorem for some Lie groups and M. Eguchi, S. Koizumi and K. Kumahara [7] also obtained the Cowling-Price theorem for motion groups. Further, J. Sengupta [13] obtained the Cowling-Price theorem on Riemannian symmetric spaces of the noncompact type.

In this paper, we prove the Cowling-Price theorem for $SU(1,1)$ under the assumption that $1 \le p, q \le \infty$ and $ab > 1/4$.

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