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General characterization theorems and intrinsic topologies in white noise analysis

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Abstract. Let u be a positive continuous function on $[0, \infty)$ satisfying the conditions: (i) $\lim_{r\to\infty} r^{-1/2}\log u(r) = \infty$, (ii) $\inf_{r\geq 0} u(r) = 1$, (iii) $\lim_{r\to\infty} r^{-1}\log u(r) < \infty$, (iv) the function $\log u(x^2)$, $x \geq 0$, is convex. A Gel'fand triple $[\mathscr{E}]_u \subset (L^2) \subset [\mathscr{E}]_u^*$ is constructed by making use of the Legendre transform of u discussed in [4]. We prove characterization theorems for generalized functions in $[\mathscr{E}]_u^*$ and for test functions in $[\mathscr{E}]_u$ in terms of their S-transforms under the same assumptions on u. Moreover, we give an intrinsic topology for the space $[\mathscr{E}]_u$ of test functions and prove a characterization theorem for measures. We briefly mention the relationship between our method and a recent work by Gannoun et al. [10]. Finally, conditions for carrying out white noise operator theory and Wick products are given.

1. Introduction

Let $\mathscr{E}$ be a real topological vector space with topology generated by a sequence of inner product norms $\{|\cdot|_p\}_{p=0}^{\infty}$. We assume that $\mathscr{E}$ is a complete metric space with respect to the metric

$$d(\xi,\eta) = \sum_{p=0}^{\infty} \frac{1}{2^p} \frac{|\xi-\eta|_p}{1+|\xi-\eta|_p}, \qquad \xi,\eta \in \mathscr{E}.$$

In addition we assume the following conditions:

(a) There exists a constant $0<\rho<1$ such that $|\cdot|_0 \leq \rho|\cdot|_1 \leq \cdots \leq \rho^p|\cdot|_p \leq \cdots$.

(b) For any $p \geq 0$, there exists $q \geq p$ such that the inclusion $i_{q,p} : \mathscr{E}_q \hookrightarrow \mathscr{E}_p$ is a Hilbert-Schmidt operator. (Here $\mathscr{E}_p$ is the completion of $\mathscr{E}$ with respect to the norm $|\cdot|_p$.)

Let $\mathscr{E}'$ and $\mathscr{E}_p'$ denote the dual spaces of $\mathscr{E}$ and $\mathscr{E}_p$, respectively. We can use the Riesz representation theorem to identify $\mathscr{E}_0$ with its dual space $\mathscr{E}_0'$. Then we get the following continuous inclusions:

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