HIROSHIMA MATH. J.
32 (2002), 145–154

On Poincaré four-complexes with free fundamental groups

Friedrich HEGENBARTH and Salvina PICCARRETA

(Received June 8, 1999)
(Revised October 23, 2001)

ABSTRACT. Let X be a closed connected and oriented PL manifold whose fundamental group π_1 is a free group of rank p. Let Λ be the integral group ring of π_1. Then $H_2(X,\Lambda)$ is Λ-free (see [6]). We show that there is a bijective correspondence between homotopy equivalence classes of 4-dimensional Poincaré complexes Y with $Y^{(3)} \cong X^{(3)}$ and invertible hermitian matrices of type k over Λ, where k is the rank of $H_2(X,\Lambda)$.

1. Introduction.

By a Poincaré 4-complex we understand a 4-dimensional CW-complex Y with a fundamental class $[Y] \in H_4(Y,\mathbf{Z}) \cong \mathbf{Z}$ inducing isomorphisms

$$\bigcap [Y] : H^q(Y,\Lambda) \to H_{4-q}(Y,\Lambda),$$

where $\Lambda = \mathbf{Z}[\pi_1(Y)]$ is the integral group ring of the fundamental group. Since we will discuss only fundamental groups which are freely generated, e.g., by p generators, there will be no Whitehead torsion. Therefore they will be finite Poincaré complexes in the sense of [8]. We can also assume that Y is obtained from the 3-skeleton $Y^{(3)}$ of Y by attaching only one 4-cell (see [8], p. 30), i.e., $Y = Y^{(3)} \cup_\varphi D^4$, where $\varphi : \mathbf{S}^3 \to Y^{(3)}$ is the attaching map. The following result of T. Matumoto and A. Katanaga will be crucial for our discussion (see [6], Proposition 2).

PROPOSITION 1.1. *Let X be a closed PL four-manifold with $\pi_1 = *^p\mathbf{Z}$. Then $X^{(3)}$ is homotopy equivalent to $\bigvee^p(\mathbf{S}^1 \vee \mathbf{S}^3) \vee (\bigvee^k \mathbf{S}^2)$.*

Let us suppose X to be a PL 4-manifold, hence $\pi_2(X) = H_2(X,\Lambda)$ is Λ-free of rank k. Recall the Whitehead exact sequence ([9])

$$0 \to \Gamma(\pi_2) \to \pi_3(X^{(3)}) \to H_3(X^{(3)},\Lambda) \to 0,$$

where $\Gamma(\pi_2)$ is the (quadratic) Γ-functor applied to the abelian group $\pi_2(X^{(3)}) = H_2(X,\Lambda)$. Note that $H_3(X^{(3)},\Lambda)$ is Λ-free of rank p. There are canonical

2000 *Mathematics Subject Classification.* 57N65, 57R67, 57Q10.
Key words and phrases. Poincaré complexes, Free fundamental groups, Homotopy type, Hermitian forms.