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Asymptotic expansion of the null distribution of the modified normal likelihood ratio criterion for testing $\Sigma = \Sigma_0$ under nonnormality

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ABSTRACT. This paper is concerned with the null distribution of the modified normal likelihood ratio criterion for testing the null hypothesis that a covariance matrix is a given one, i.e., $\Sigma = \Sigma_0$, under nonnormality. We obtain an asymptotic expansion of the null distribution of the test statistic up to the order n^{-1}, where n is the sample size, under nonnormality by using an Edgeworth expansion of the density function of a sample covariance matrix.

1. Introduction

Let $\boldsymbol{x}_1, \dots, \boldsymbol{x}_n$ be $p \times 1$ random vevtors, where n is the sample size. It is assumed that each vector $\boldsymbol{x}_j$ is $i.i.d.$ with the mean $\mathrm{E}(\boldsymbol{x}) = \boldsymbol{\mu}$ and the covariance matrix $\mathrm{Cov}(\boldsymbol{x}) = \Sigma$. Consider testing the null hypothesis that the covariance matrix is a given one, i.e.,

$$H_0 : \Sigma = \Sigma_0. \tag{1.1}$$

Then a commonly used test statistic is

$$T = -2 \log L, \tag{1.2}$$

which is a modified likelihood ratio statistic for a multivariate normal population, where

$$L = \left(\frac{e}{n-1}\right)^{p(n-1)/2} |S\Sigma_0^{-1}|^{(n-1)/2} \exp\left\{-\frac{1}{2} \operatorname{tr}(S\Sigma_0^{-1})\right\},$$

$$S = \sum_{j=1}^{n} (\boldsymbol{x}_j - \bar{\boldsymbol{x}})(\boldsymbol{x}_j - \bar{\boldsymbol{x}})', \qquad \bar{\boldsymbol{x}} = \frac{1}{n}\sum_{j=1}^{n} \boldsymbol{x}_j.$$

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