HIROSHIMA MATH. J.
33 (2003), 379–389

A note on the Hopf homomorphism of a Toda bracket and its application

Tomohisa INOUE and Juno MUKAI*

(Received May 22, 2003)

ABSTRACT. The purpose of the present note is to extend a formula between the Toda bracket and Hopf homomorphism. As an application, we show that the generator of the 2-primary component of the homotopy group $\pi_{12}(S^5)$ is taken as a representative of a specific Toda bracket. And we shall give a short proof of the existence of the unstable Adams map.

1. Introduction

In this note all spaces, maps and homotopies are based. For a space X, we denote by ΣX a suspension of X and by $X \wedge X$ a smash product of X and itself. The Toda bracket [8] has some properties relative to the Hopf homomorphism $H : [\Sigma K, S^{n+1}] \to [\Sigma K, S^{2n+1}]$, where K is a CW-complex and S^n is the n-sphere. We shall extend the n-sphere to a CW-complex with exactly one vertex, that is, we define a generalized Hopf homomorphism $H : [\Sigma K, \Sigma A] \to [\Sigma K, \Sigma(A \wedge A)]$ for a CW-complex A with one vertex and prove the properties between the Toda bracket and this Hopf homomorphism. Then we can apply them to spaces of suspensions of the real projective plane and the quasi-quaternionic projective plane. From this extension, we find out a roundabout approach to determine the 2-primary component π_{14}^7 of the homotopy group $\pi_{14}(S^7)$ [8]. And we shall prove a short and intuitive proof of the existence of the unstable Adams map [6], [3].

The authors wish to thank Professors Kachi, Matsuda and Tamaki for useful comments and advices.

2. Generalizations of Toda formulas

We shall recall the James construction [1]. Let A be a Hausdorff space. Denote by A_∞ the reduced product space of A [1]. Let a_0 be the base point of

2000 *Mathematics Subject Classification*. Primary 55Q25; Secondary 55Q25, 55Q40.

Key words and phrases. Hopf homomorphism, Toda bracket.

* Partially supported by Grant-in-Aid for Scientific Research (No. 15540067 (c), (2)), Japan Society for the Promotion of Science.