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Stable extendibility of vector bundles over real projective spaces and bounds for the Schwarzenberger numbers $\beta(k)$

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ABSTRACT. For a non-negative integer k, R. L. E. Schwarzenberger defined in [**7**] an integer $\beta(k) \geq 0$ which we call the Schwarzenberger number of k. Let ζ be a k-dimensional F-vector bundle over the real projective n-space RP^n, where F is either the real number field R or the complex number field C. Then $\beta(k)$ is closely related to the problem to find the dimension m with $m \geq n$ which has the property that ζ is stably equivalent to a sum of k F-line bundles if ζ is stably extendible to RP^m. The problem for $F = R$ has been studied in [**7**], [**5**] and [**4**], and that for $F = C$ has been studied in [**6**] and [**4**]. In this note we obtain further results on the problem and determine bounds for the Schwarzenberger numbers $\beta(k)$.

1. Introduction

Throughout this note, F denotes either the real number field R or the complex number field C, and N is the set of all non-negative integers. Let X be a space and A its subspace. A k-dimensional F-vector bundle ζ over A is said to be *extendible* (respectively *stably extendible*) to X, if there is a k-dimensional F-vector bundle over X whose restriction to A is equivalent (respectively stably equivalent) to ζ, that is, if ζ is equivalent (respectively stably equivalent) to the induced bundle $i^*\eta$ of a k-dimensional F-vector bundle η over X under the inclusion map $i : A \to X$ (cf. [**7**, p. 20], [**8**, p. 191] and [**3**, p. 273]).

For a positive integer i, write $i = (2a+1)2^{\nu(i)}$, where $a \in N$, and for $k \in N$ define an integer $\beta(k) \in N$ by

$$\beta(k) = \min\{i - \nu(i) - 1 \mid k < i\}$$

which we call the Schwarzenberger number of k.

Let ζ be a k-dimensional F-vector bundle over the real projective n-space RP^n where $k > 0$. We study the problem to find the dimension m with $m \geq n$ which has the property that ζ is stably equivalent to a sum of k F-line bundles if ζ is stably extendible to RP^m.

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