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Subgroups of $\pi_*(L_2T(1))$ at the prime two

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ABSTRACT. Let $T(1)$ be the Ravenel spectrum whose BP_*-homology is $BP_*[t_1](\subset BP_*(BP))$, and let L_2 denote the Bousfield localization functor with respect to $v_2^{-1}BP$. In this paper, we show that the E_4-term of the Adams-Novikov spectral sequence for $\pi_*(L_2T(1))$ has horizontal vanishing line and is the E_∞-term. We also find subgroups of the homotopy groups $\pi_*(L_2T(1))$.

1. Introduction

In this paper, everything is localized at the prime two. Let BP denote the Brown-Peterson ring spectrum at the prime two. Then the homotopy groups $\pi_*(BP)$ turn to the polynomial algebra $BP_* = \mathbf{Z}_{(2)}[v_1, v_2, \dots]$ over the Hazewinkel generators v_i with $|v_i| = 2^{i+1} - 2$. The Ravenel spectrum $T(1)$ is characterized by the Brown-Peterson homology as $BP_*(T(1)) = BP_*[t_1] \subset BP_*(BP) = BP_*[t_1, t_2, \dots]$. We consider the spectrum $G = v_2^{-1}BP$. Let L_2 denote the Bousfield localization functor on the stable homotopy category of spectra with respect to G. One of the methods to determine the homotopy groups $\pi_*(L_2T(1))$ is the Adams-Novikov spectral sequence $E_2^* = H^*v_2^{-1}BP_*[t_1] \Rightarrow \pi_*(L_2T(1))$, where $H^*- = \operatorname{Ext}^*_{G_*(G)}(G_*, -)$. We study the E_2-term by the chromatic spectral sequence $\sum_{i=0}^2 H^*M_0^i[t_1] \Rightarrow H^*v_2^{-1}BP_*[t_1]$ and the mod 2 Bockstein spectral sequences $H^*M_1^0[t_1] \Rightarrow H^*M_0^1[t_1]$ and $H^*M_1^1[t_1] \Rightarrow H^*M_0^2[t_1]$. Here, $M_0^0 = 2^{-1}BP_*$, $M_1^0 = v_1^{-1}BP_*/(2)$, $M_0^1 = v_1^{-1}BP_*/(2^\infty)$, $M_1^1 = v_2^{-1}BP_*/(2, v_1^\infty)$ and $M_0^2 = v_2^{-1}BP_*/(2^\infty, v_1^\infty)$. The modules $H^*M_0^0[t_1]$ and $H^*M_1^0[t_1]$ are given by Ravenel in [7]. In [5], Mahowald and the second author determined $H^*M_2^0[t_1]$ as the tensor product of the polynomial algebra $K(2)_*[v_3, h_{20}]$ and the exterior algebra $\Lambda(h_{21}, h_{30}, h_{31}, \rho_2)$, where $K(2)_* = \mathbf{Z}/2[v_2^{\pm 1}]$. In [8], the second author determined $H^*M_1^1[t_1]$ by the v_1-Bockstein spectral sequence $H^*M_2^0[t_1] \Rightarrow H^*M_1^1[t_1]$ to be the tensor product of $\Lambda(\rho_2)$ and the direct sum of modules A_i:

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