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Stable unextendibility of vector bundles over the quaternionic projective spaces

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ABSTRACT. We study the stable unextendibility of vector bundles over the quaternionic projective space $\mathbf{H}P^n$ by making use of combinatorial properties of the Stiefel-Whitney classes and the Pontrjagin classes. First, we show that the tangent bundle of $\mathbf{H}P^n$ is not stably extendible to $\mathbf{H}P^{n+1}$ for $n \geq 2$, and also induce such a result for the normal bundle associated to an immersion of $\mathbf{H}P^n$ into $\mathbf{R}^{4n+k}$. Secondly, we show a sufficient condition for a quaternionic r-dimensional vector bundle over $\mathbf{H}P^n$ not to be stably extendible to $\mathbf{H}P^{n+l}$ for $r \leq n$ and $l > 0$, which is also a necessary condition when $r = n$ and $l = 1$.

1. Introduction and results

Let F be the real field $\mathbf{R}$, the complex field $\mathbf{C}$ or the quaternionic skew field $\mathbf{H}$. Then, an F-vector bundle V of dimension k over a base space B is called extendible to a space B' with $B \subset B'$ if there exists an F-vector bundle W of dimension k over B' whose restriction to B is isomorphic to V as F-vector bundles. That is, $i^*W \cong V$ for the inclusion map $i : B \to B'$. If i^*W is stably equivalent to V, namely $i^*W + m_F \cong V + m_F$ for a trivial F-vector bundle m_F of dimension $m \geq 0$, then V is called stably extendible to B' ([6]).

It is an interesting problem to determine when given vector bundles are stably extendible or not, which is related to some stable properties of vector bundles like geometrical dimensions or decompositions to line bundles (cf. [12], [2], [11], [9]). In this paper, we study the stable unextendibility of some vector bundles over the quaternionic projective space $\mathbf{H}P^n$.

Schwarzenberger ([4, Appendix 1]) has shown, as an application of the Riemann-Roch theorem, that the tangent bundle $T(\mathbf{C}P^n)$ of the complex projective space $\mathbf{C}P^n$ for $n \geq 2$ is not extendible to $\mathbf{C}P^{n+1}$ as $\mathbf{C}$-vector bundle. Kobayashi-Maki-Yoshida [7] has also shown that the tangent bundle $T(\mathbf{R}P^n)$ (resp. $T(L^n(p))$) of the real projective space $\mathbf{R}P^n$ (resp. the lens space $L^n(p)$ for

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