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Oscillation and nonoscillation theorems for a class of fourth order quasilinear functional differential equations

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ABSTRACT. The oscillatory behavior of fourth order functional differential equations

$$(\mathrm{A}) \qquad (|y''(t)|^{\alpha}\ \mathrm{sgn}\ y''(t))'' + q(t)|y(g(t))|^{\beta}\ \mathrm{sgn}\ y(g(t)) = 0$$

is investigated. First, criteria are given for the existence of nonoscillatory solutions with specific asymptotic behavior, and then criteria for all solutions to be oscillatory are derived by comparing (A) with the associated differential equation without functional argument.

1. Introduction

The objective of this paper is to study the oscillatory and nonoscillatory behavior of fourth order nonlinear functional differential equations

$$(\mathrm{A}) \qquad (|y''(t)|^{\alpha}\ \mathrm{sgn}\ y''(t))'' + q(t)|y(g(t))|^{\beta}\ \mathrm{sgn}\ y(g(t)) = 0$$

where

(a) α and β are positive constants;
(b) $q : [0, \infty) \to (0, \infty)$ is a continuous function;
(c) $g : [0, \infty) \to (0, \infty)$ is a continuously differentiable function such that $g'(t) > 0,\ t \geq 0,$ and $\lim_{t\to\infty} g(t) = \infty$.

By a solution of (A) we mean a function $y : [T_y, \infty) \to \mathbf{R}$ which is twice continuously differentiable together with $|y''|^{\alpha}\ \mathrm{sgn}\ y''$ and satisfies the equation (A) at all sufficiently large t. Those solutions which vanish in a neighborhood of infinity will be excluded from our consideration. A solution is said to be oscillatory if it has a sequence of zeros clustering around ∞, and nonoscillatory otherwise.

We first (in Section 1) study the existence of nonoscillatory solutions. The set of nonoscillatory solutions of (A) is decomposed into six disjoint classes according to their asymptotic behavior at ∞, and existence criteria are established for each of these classes. Some of the criteria are shown to be necessary as well.

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