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Behavior of the life span for solutions to the system of reaction-diffusion equations

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Abstract. We consider the weakly coupled system of reaction-diffusion equations[1,2]

$$u_t = \Delta u + a(x)v^p, \qquad v_t = \Delta v + b(x)u^q,$$

$$u(x,0) = \lambda^{\mu}\varphi(x), \qquad v(x,0) = \lambda^{\nu}\psi(x),$$

where $0 \le a(x)$, $b(x) \in C(\mathbf{R}^N)$, $\varphi(x), \psi(x) \ge 0$ are bounded continuous functions in $\mathbf{R}^N$, $p, q > 1$, $\mu, \nu > 0$, and $\lambda > 0$ are parameters. The existense of solutions, blow-up conditions, and global solutions of the above equations with $a(x) \equiv |x|^{\sigma_1}$, $b(x) \equiv |x|^{\sigma_2}$ $(0 \le \sigma_1 < N(p-1),\ 0 \le \sigma_2 < N(q-1))$ are studied by Mochizuki and Huang. In this paper, we consider an estimate of maximal existence time of blow-up solutions as λ goes to 0 or ∞, when $a(x), b(x)$ are more general functions.

1. Introduction and statement of results

We consider bounded, nonnegative solutions to the Cauchy problem for a weakly coupled system

$$\begin{cases} u_t = \Delta u + a(x)v^p & (x \in \mathbf{R}^N, t > 0), \\ v_t = \Delta v + b(x)u^q & (x \in \mathbf{R}^N, t > 0), \\ u(x,0) = \lambda^{\mu}\varphi(x) & (x \in \mathbf{R}^N), \\ v(x,0) = \lambda^{\nu}\psi(x) & (x \in \mathbf{R}^N), \end{cases} \tag{1}$$

where $0 \le a(x)$, $b(x) \in C(\mathbf{R}^N)$, $0 \le \varphi(x)$, $\psi(x) \in BC(\mathbf{R}^N)$; here $BC(\mathbf{R}^N)$ is the set of bounded continuous functions on $\mathbf{R}^N$, $p, q > 1$, $\mu, \nu > 0$, and $\lambda > 0$ are parameters. Since the nonlinearities, $a(x)v^p, b(x)u^q$, are locally continuous in x and locally Lipschitz in u, v, it follows from standard results that any solution $u(x,t), v(x,t) \ge 0$ of the equation (1) is in fact classical; that is, $u, v \in C^{2,1}(\mathbf{R}^N \times (0,T)) \cap C(\mathbf{R}^N \times [0,T))$ for some $T > 0$. Thus, the comparison theorem holds from Theorem 1 in [1]; i.e. if

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[1] 2000 *Mathematics Subject Classification*. 35K57, 58J35.

[2] *Key Words and Phrases*. reaction-diffusion equations, heat and other parabolic equation methods.