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# Longitudinal slope and Dehn fillings

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**ABSTRACT.** Let $M$ be an irreducible 3-manifold with an incompressible torus boundary $T$, and $\gamma$ a slope on $T$, which bounds an incompressible surface, with genus $g$ say. We assume that there exists a slope $r$ that produces an essential 2-sphere by Dehn filling.

Let $q$ be the minimal geometric intersection number between the essential 2-sphere and the core of the Dehn filling. Then, we show that $q = 2$ or the minimal geometric intersection number between $\gamma$ and $r$ is bounded by $2g - 1$.

In the special case that $M$ is the exterior of a non-cable knot $K$ in $S^3$, we show that $q \geq 6$ and $|r| \leq 2g - 1$, where $g$ is the genus of the knot $K$. We get also similar and simpler results for the projective slopes. These imply immediately a known result that the cabling and $\mathbf{R}P^3$ conjectures are true for genus one knots.

## 1. Introduction

All 3-manifolds are assumed to be compact and orientable. Let $M$ be a 3-manifold, with a torus $T$ as boundary. *A slope* $r$ on $T$ is the isotopy class of an unoriented essential simple closed curve on $T$. The slopes are parametrized by $\mathbf{Q} \cup \{\infty\}$ (for more details, see [25]).

A *Dehn filling on* $M$ is to glue a solid torus $V = S^1 \times D^2$ to $M$ along $T$. We call it an $r$-*Dehn filling* when the attaching homeomorphism sends a meridian curve of $\partial V$ to the slope $r$ on $T$. We denote by $M(r)$ the resulting 3-manifold after the $r$-Dehn filling.

A 3-manifold is *reducible* if it contains an essential 2-sphere, that is, a 2-sphere which does not bound a 3-ball; otherwise it is an *irreducible* 3-manifold. A slope $r$ in $T$ is said to be a *reducing slope* if $M$ is irreducible and $M(r)$ is reducible (that means that $r$ produces an essential 2-sphere).

Similarly, a *projective slope* is a slope $p$ that produces a projective plane by Dehn filling. This means that $M$ does not contain a projective plane but $M(p)$ contains a projective plane.

Many papers focus on projective or reducing slopes:

i) There exist at most three reducing slopes (see [15, 19]) and three projective slopes (see [22, 28]);

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