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Asymptotic expansion of the null distribution of the likelihood ratio statistic for testing the equality of variances in a nonnormal one-way ANOVA model

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ABSTRACT. This paper is concerned with the null distribution of the likelihood ratio statistic for testing the equality of variances of q nonnormal populations. It is known that the null distribution of this statistic converges to χ^2_{q-1} under normality. We extend this result by obtaining an asymptotic expansion under general conditions. Numerical accuracies are studied for some approximations of the percentage points and actual test sizes of this statistic based on the limiting distribution and the asymptotic expansion.

1. Introduction

The one-way ANOVA test is a familiar procedure for comparing several populations. Let X_{ij} be the j-th sample observation $(j = 1, \ldots, n_i)$ from the i-th population Π_i $(i = 1, \ldots, q)$ with mean μ_i and common variance σ^2, where μ_i's and σ^2 are unknown. The null hypothesis which is considered in this test is $H_0 : \mu_1 = \cdots = \mu_q$. Let $n = n_1 + \cdots + n_q$, $\overline{X}_i = n_i^{-1} \sum_{j=1}^{n_i} X_{ij}$ and $\overline{X} = n^{-1} \sum_{i=1}^{q} \sum_{j=1}^{n_i} X_{ij}$. A commonly used statistic is $T = (n-q)S_h/S_e$, which is the likelihood ratio statistic for the normal case, where $S_h = \sum_{i=1}^{q} n_i(\overline{X}_i - \overline{X})^2$, $S_e = \sum_{i=1}^{q}(n_i - 1)s_i^2$ and $s_i^2 = (n_i - 1)^{-1} \sum_{j=1}^{n_i}(X_{ij} - \overline{X}_i)^2$. Under normality, i.e., $\Pi_i : \mathrm{N}(\mu_i, \sigma^2)$, it is well known that the null distribution of $(q-1)^{-1}T$ is distributed as F_{n-q}^{q-1}. Under nonnormality, it is known that the null distribution of this statistic converges to χ^2_{q-1} and an asymptotic expansion of the null distribution was obtained by Fujikoshi, Ohmae and Yanagihara (1999). Under normality, it is known that this test is robust against heteroscedasticy of the variances and under nonnormality an asymptotic expansion of the null distribution of the test statistic, proposed by James (1951), was obtained by Yanagihara (2000). As thses tests depend on the assumption of variances, it is important to test the equality of variances as a preliminary to one-way ANOVA test.

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