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Differentiability properties of some nonlinear operators associated to the conformal welding of Jordan curves in Schauder spaces

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Abstract. As it is well-known, to a given plane simple closed curve ζ with non-vanishing tangent vector, one can associate a conformal welding homeomorphism $\mathbf{w}[\zeta]$ of the unit circle to itself, obtained by composing the restriction to the unit circle of a suitably normalized Riemann map of the domain exterior to ζ with the inverse of the restriction to the unit circle of a suitably normalized Riemann map of the domain interior to ζ. Now we think the functions ζ and $\mathbf{w}[\zeta]$ as points in a Schauder function space on the unit circle, and we show that the correspondence $\mathbf{w}$ which takes ζ to $\mathbf{w}[\zeta]$ is real differentiable for suitable exponents of the Schauder spaces involved. Then we show that $\mathbf{w}$ has a right inverse which is the restriction of a holomorphic nonlinear operator.

1. Introduction

As it is well-known, given an element ζ of the set $\mathscr{A}_{\partial\mathbf{D}}$ of the complex-valued differentiable injective functions, with nonvanishing first derivative, defined on the boundary $\partial\mathbf{D}$ of the open unit disk $\mathbf{D}$ of the complex plane $\mathbf{C}$, the function ζ parametrizes a Jordan curve. To each $\zeta \in \mathscr{A}_{\partial\mathbf{D}}$, one can associate a pair (G, F) of Riemann maps, with G a suitably normalized holomorphic homeomorphism of the exterior $\mathbf{C}\backslash \mathrm{cl}\,\mathbf{D}$ of $\mathbf{D}$ onto the exterior $E[\zeta]$ of ζ, and with F a suitably normalized holomorphic homeomorphism of $\mathbf{D}$ onto the interior $I[\zeta]$ of ζ. It is also well-known that G and F can be extended with continuity to boundary homeomorphisms. Thus one can consider the so-called conformal welding homeomorphism $F^{(-1)} \circ G_{|\partial\mathbf{D}}$ of $\partial\mathbf{D}$, which we denote by $\mathbf{w}[\zeta]$. Now let $C_*^{m,\alpha}(\partial\mathbf{D}, \mathbf{C})$ be the Schauder space of m-times continuously differentiable complex-valued functions on $\partial\mathbf{D}$, whose m-th order derivative is α-Hölder continuous, with $\alpha \in]0,1[$, $m \geq 1$. It is well-known that if $\zeta \in C_*^{m,\alpha}(\partial\mathbf{D}, \mathbf{C}) \cap \mathscr{A}_{\partial\mathbf{D}}$, then $\mathbf{w}[\zeta] \in C_*^{m,\alpha}(\partial\mathbf{D}, \mathbf{C}) \cap \mathscr{A}_{\partial\mathbf{D}}$. In this paper we first prove some differentiability theorems for the nonlinear 'conformal welding operator' $\mathbf{w}[\cdot]$. We note that such theorems can be shown to be optimal in the frame of Schauder spaces (cf. [19, Thm. 2.14].) Moreover,

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