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Removability of sets for sub-polyharmonic functions

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Abstract. Our first aim in this paper is to generalize Bôcher's theorem for functions u whose Riesz measure $\mu = \Delta^m u$ is nonnegative in the punctured unit ball $\mathbf{B}_0$. In fact, if u satisfies a certain integral condition and $\mu = \Delta^m u \geq 0$ in $\mathbf{B}_0$, then it is shown that u can be written as the sum of a generalized potential of μ and a polyharmonic function on $\mathbf{B}$. This is nothing but the Laurent series expansion for u.

The next aim is to give a polyharmonic version of the recent results by Riihentaus [11] concerning removability of sets for subharmonic functions.

1. Introduction and statement of results

Let $\mathbf{R}^n$ be the n-dimensional Euclidean space with a point $x = (x_1, x_2, \dots, x_n)$. For a multi-index $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_n)$, we set

$$|\lambda| = \lambda_1 + \lambda_2 + \cdots + \lambda_n,$$

$$x^\lambda = x_1^{\lambda_1} x_2^{\lambda_2} \dots x_n^{\lambda_n}$$

and

$$D^\lambda = \left(\frac{\partial}{\partial x_1}\right)^{\lambda_1} \left(\frac{\partial}{\partial x_2}\right)^{\lambda_2} \cdots \left(\frac{\partial}{\partial x_n}\right)^{\lambda_n}.$$

We denote by $B(x, r)$ the open ball centered at x with radius $r > 0$, whose boundary is written as $S(x, r) = \partial B(x, r)$. We also denote by $\mathbf{B}$ the unit ball $B(0, 1)$ and by $\mathbf{B}_0$ the punctured unit ball $\mathbf{B} - \{0\}$.

A real-valued function u on an open set $G \subset \mathbf{R}^n$ is called polyharmonic of order m on G if $u \in C^{2m}(G)$ and $\Delta^m u = 0$ on G, where m is a positive integer, Δ denotes the Laplacian and $\Delta^m u = \Delta^{m-1}(\Delta u)$ (cf. [2], [10]). We denote by $H^m(G)$ the space of polyharmonic functions of order m on G. In particular, u is harmonic on G if $u \in H^1(G)$.

The fundamental solution of Δ^m is written as R_{2m}, that is,

$$R_{2m}(x) = \begin{cases} \alpha_m |x|^{2m-n} & \text{if } 2m - n \text{ is not an even nonnegative integer,} \\ \alpha_m |x|^{2m-n} \log(1/|x|) & \text{if } 2m - n \text{ is an even nonnegative integer,} \end{cases}$$

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