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Adjustment on an asymptotic expansion of the distribution function with χ^2-approximation

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ABSTRACT. This paper is concerned with an adjustment on an asymptotic expansion of the distribution function, whose limiting distribution is a chi-squared distribution, up to the order n^{-1}. The distribution function is a monotone function and has the upper and lower bounds with 0 and 1, but an asymptotic expansion does not satisfy these properties. We consider to add a term of n^{-2} order to the asymptotic expansion so that the resulting one satisfies such properties. Note that our adjustment does not give an influence on the order of the remainder term in the asymptotic expansion. Our method of preserving monotoneity is based on the idea in Kakizawa (1996).

1. Introduction

The limiting distribution is often used as an approximate distribution, when it is difficult to obtain the exact distribution function for its compexity. However, its accuracy tends to be bad as the sample size n tends to be small. It is well known that an asymptotic expansion will improve accuracy of approximation compared with the limiting distribution in the small sample case. Suppose that a nonnegative random variate T has an asymptotic expansion such that

$$\begin{aligned} \mathrm{P}(T \le x) &= G_r(x) + \frac{1}{n}\sum_{j=0}^{k} b_j G_{r+2j}(x) + \mathrm{o}(n^{-1}) \\ &= P_{ae}(x) + \mathrm{o}(n^{-1}), \end{aligned} \tag{1.1}$$

where $G_r(x)$ is the distribution function of a central χ^2 distribution with r degrees of freedom, coefficients b_j's satisfy the relation $\sum_{j=0}^{k} b_j = 0$ and k is a certain positive integer. The approximation $P_{ae}(x)$ with the supplementary term $\sum_{j=0}^{k} b_j G_{r+2j}(x)/n$ will give a better approximation than the limiting dis-

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