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Classification of the incompressible spanning surfaces for prime knots of 10 or less crossings

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ABSTRACT. It is known that the incompressible spanning surfaces for a fibred knot are unique. Also for a 2-bridge knot its incompressible spanning surfaces were classified by Hatcher and Thurston. In this paper we shall give the classification of the incompressible spanning surfaces for prime knots of 10 or less crossings, which include many non-fibred and non-2-bridge knots. Furthermore, we determine the associated simplicial complex $IS(K)$ for each prime knot K of 10 or less crossings, which describes the relations between equivalence classes of incompressible spanning surfaces for K.

Introduction

It is known that the incompressible spanning surfaces for a fibred knot are unique in the sense stated below (cf. [17]). Also for a 2-bridge knot its incompressible spanning surfaces were classified by Hatcher and Thurston [8]. In this paper we shall give the classification of the incompressible spanning surfaces for prime knots of 10 or less crossings which include many non-fibred and non-2-bridge knots. Furthermore, we determine the *associated simplicial complex* $IS(K)$ for each prime knot K of 10 or less crossings, which was introduced in [11] to describe the relations between equivalence classes of incompressible spanning surfaces for K.

Let L be an oriented link in the 3-sphere S^3, and let $E(L) = S^3 - \mathrm{Int}\, N(L)$ be its exterior where $N(L)$ is a fixed tubular neighborhood of L. We shall use the term "spanning surface" for L to denote a surface $S = \Sigma \cap E(L)$ where Σ is an oriented surface in S^3 such that $\partial\Sigma = L$, Σ has no closed component and is possibly disconnected and that $\Sigma \cap N(L)$ is a collar of $\partial\Sigma$ in Σ. Two spanning surfaces for L are said to be *equivalent* if they are ambient isotopic in $E(L)$ to each other. A spanning surface S is *incompressible* (resp. *of minimal genus*) if each component of S is π_1-injective in $E(L)$ (resp. the Euler number $\chi(S)$ is maximum among all the spanning surfaces for L). In this paper "link" always means *oriented link*. If L is a knot, then the classification of the incompres-

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