Analogues of sampling theorems for some homogeneous spaces

Mitsuhiko Ebata, Masaaki Eguchi*, Shin Koizumi and
Keisaku Kumahara[†]

Abstract. Sampling theorems are one of the basic tools in information theory. The signal function f whose band–region is contained in a certain interval can be reconstructed from their values $f(x_k)$ at the sampling points $\{x_k\}$. We obtain analogues of this theorem for the cases of the Fourier–Jacobi series, the complex sphere S_c^{n-1} and the complex semisimple Lie groups. And as an application of these formulae, we show a version of the sampling theorem for the Radon transform on the complex hyperbolic space.

1. Introduction

Sampling theorems are one of the basic tools in information theory and various types of sampling theorems are obtained in many papers. The Shannon sampling theorem is well known as a fundamental tool. A signal function is called to be band–limited if its band–region is contained in a certain interval. In the terminology of Fourier analysis, the band–limitedness condition is equivalent to the condition that the support of the Fourier transform $\tilde{f}$ of $f \in L^2(\mathbf{R})$ is contained in a certain interval. The Shannon sampling theorem yields that if a function $f \in L^2(\mathbf{R})$ is band–limited, then f can be reconstructed by samples taken at the equidistant sampling points. We are interested in generalizing sampling theorems to the cases of homogeneous spaces. In this paper, we obtain analogues of this theorem in the cases of the Fourier–Jacobi series, the complex sphere $S_c^{n-1} = U(n)/U(n-1)$ and the complex semisimple Lie groups.

On the other hand, the problem how to recover the values of the functions from the samples of their Radon transforms is studied in the theory of the