Global behavior of two-dimensional difference equations system with two period coefficients

Merve Kara[1], Durhasan Turgut Tollu[2] and Yasin Yazlik[3,*]

[1]Aksaray University, Ortaköy Vocational High School, Aksaray, Turkey
[2]Necmettin Erbakan University, Faculty of Science, Department of Mathematics, Konya, Turkey
[3]Nevsehir Hacı Bektaş Veli University, Faculty of Science and Art, Department of Mathematics, Nevsehir, Turkey
E-mail: yyazlik@nevsehir.edu.tr*

Abstract

In this paper, we investigate the following system of difference equations

$$x_{n+1}=\frac{\alpha_n}{1+y_nx_{n-1}},\ y_{n+1}=\frac{\beta_n}{1+x_ny_{n-1}},\ n\in\mathbb{N}_0,$$

where the sequences $(\alpha_n)_{n\in\mathbb{N}_0}$, $(\beta_n)_{n\in\mathbb{N}_0}$ are positive, real and periodic with period two and the initial values x_{-1}, x_0, y_{-1}, y_0 are non-negative real numbers. We show that every positive solution of the system is bounded and examine their global behaviors. In addition, we give closed forms of the general solutions of the system by using the change of variables. Finally, we present a numerical example to support our results.

2000 Mathematics Subject Classification. **39A10**. 39A20, 39A23.
Keywords. global behavior, solution in closed form, system of difference equations.

1 Introduction

As a prototype, in [1], Drymonis investigated the global stability, periodic character and boundedness of solution of the following difference equation by distinguishing several special cases

$$x_{n+1}=\frac{\alpha_n+\beta_nx_nx_{n-1}+\gamma_nx_{n-1}}{A_n+B_nx_nx_{n-1}+C_nx_{n-1}},\quad n\in\mathbb{N}_0,\tag{1}$$

where the parameters α_n, β_n, γ_n, A_n, B_n, C_n are non-negative periodic sequences and the initial values x_{-1}, x_0 are non-negative real numbers. In [2–5], equation (1) with constant coefficients is studied in the global stability, periodic and boundedness of solutions of some particular cases. Kulenovic et al, obtained five equations for the related of equation (1) with constant coefficients in [5]. Moreover, Amleh et al. studied thirty equations which are special case of equation (1) and constant coefficients in [2, 3]. One of thirty equations considered in [2] is the rational difference equation given as follows:

$$x_{n+1}=\frac{\alpha}{1+x_nx_{n-1}},\ n\in\mathbb{N}_0.\tag{2}$$

Further, some featured studies on the stability of the particular cases with constant coefficients of equation (1) can be found in the literature, (see, [6–11]). On the other hand, many authors obtain some closed-form formulas which are solutions special cases of the equation (1) in [4, 12–19]. The interesting thing is that all of them have constant coefficients. Equation (1) is extended to the two-dimensional and the three-dimensional systems of difference equation with constant coefficients

Tbilisi Mathematical Journal 13(4) (2020), pp. 49–64.
Tbilisi Centre for Mathematical Sciences.

Received by the editors: 20 March 2020.
Accepted for publication: 18 September 2020 .